# Strategic games and equilibrium concepts

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  - If one confesses and the other remains silent, then the former will be set free and the latter will go to prison for 5 years

# Prisoner's dilemma

- We can represent their payoffs using a bi-matrix

| | confess | silent |
|---|---|---|
| **confess** | **-3, -3** | **0, -5** |
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  - **best action = confess**

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- How does the row-prisoner think in order to find the best action?

- In any case, **confessing is the best action**, and the same holds for the column-prisoner due to symmetry
- Confessing is a **dominant strategy** for both prisoners since, whatever the other prisoner does, this action is always better

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  - Such a strategy is called a **best response**

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- Each **state of the game** (defined by a strategy per player) yields a **payoff** (or **utility**) to each player
- Given the strategies of the other players, each player aims to select its strategy in order to maximize its utility
  - Such a strategy is called a **best response**
- A state consisting of best responses is stable, and called a **pure Nash equilibrium**: no player would like to deviate and select a different strategy

# Back to prisoner's dilemma

- Players = the two prisoners
- Strategies = {confess, silent}
- Possible states = {(confess, confess), (confess, silent), (silent, confess), (silent, silent)}
- Utilities given by the bi-matrix:

| | confess | silent |
|---|---|---|
| confess | -3, -3 | 0, -5 |
| silent | -5, 0 | -1, -1 |

- Confessing is a best response to any strategy of the other player
- (confess, confess) is a pure Nash equilibrium of the game

# Battle of the sexes

- A couple (man and woman) want to decide what to do this evening; they can either attend a sports game or stay home and watch a movie
- They have different utilities for the two activities, but they would like to be together

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| | | man | |
|---|---|---|---|
| | | **sports** | **movie** |
| woman | **sports** | **3, 6** | **1, 1** |
| | **movie** | **2, 2** | **6, 3** |

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- How does the woman think?

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- How does the woman think?
- If the man chooses sports, then she also prefers sports (3 vs. 2)

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| | | man | |
|---|---|---|---|
| | | **sports** | **movie** |
| woman | **sports** | **3, 6** | **1, 1** |
| | **movie** | **2, 2** | **6, 3** |

- How does the woman think?
- If the man chooses sports, then she also prefers sports (3 vs. 2)
- If the man chooses movie, then she also prefers movie (6 vs. 1)

# Battle of the sexes

| | | man: sports | man: movie |
|---|---|---|---|
| woman | sports | 3, 6 | 1, 1 |
| | movie | 2, 2 | 6, 3 |

- There is **no dominant strategy** for the woman (nor for the man)
- What is the equilibrium strategy profile then?

# Battle of the sexes

| | | man | |
|---|---|---|---|
| | | **sports** | **movie** |
| woman | **sports** | **3, 6** | **1, 1** |
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- Is the state (movie, sports) an equilibrium?

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| | | **sports** | **movie** |
| woman | **sports** | **3, 6** | **1, 1** |
| | **movie** | **2, 2** | **6, 3** |

- Is the state (movie, sports) an equilibrium?
- No, the woman would prefer to **unilaterally change** her strategy to sports:
  - the state (sports, sports) gives her utility 3, while now she only gets utility 2

# Battle of the sexes

| | | man | |
|---|---|---|---|
| | | **sports** | **movie** |
| woman | **sports** | **3, 6** | **1, 1** |
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| woman | **sports** | **3, 6** | **1, 1** |
| | **movie** | **2, 2** | **6, 3** |

- Is the state (sports, sports) an equilibrium?
- Yes, none of the two players has incentive to unilaterally change its strategy:
  - a deviation to movie would give utility 1 to the man and 2 to the woman, compared to the utility of 6 and 3 they now get

# Nash dynamics graph

- An easy way to graphically find Nash equilibria
- Built a graph containing a node per state
- A directed edge between two nodes represents the fact that there exists a player with a profitable unilateral deviation
- A node with only incoming edges corresponds to an equilibrium state: no player would like to deviate from there

# Battle of the sexes

| | | man | |
|---|---|---|---|
| | | sports | movie |
| woman | sports | 3, 6 | 1, 1 |
| | movie | 2, 2 | 6, 3 |

sports, sports

sports, movie

movie, sports

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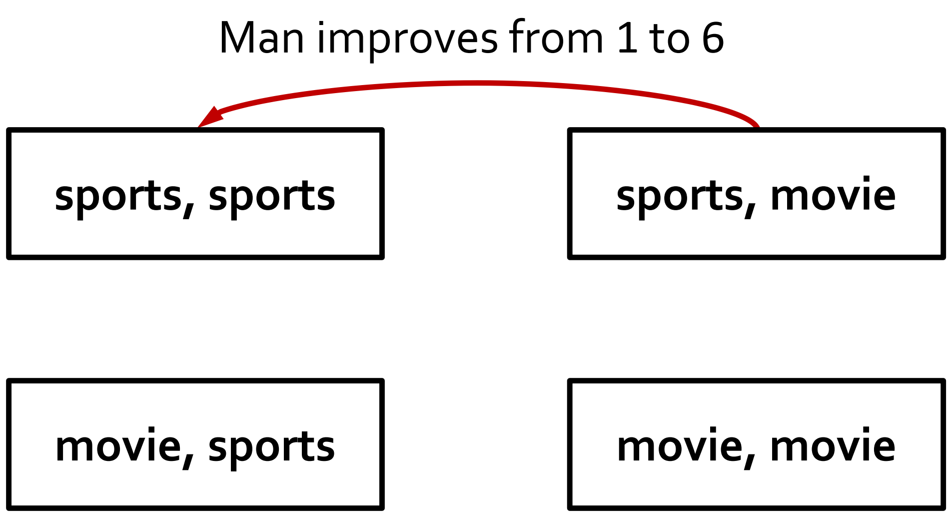

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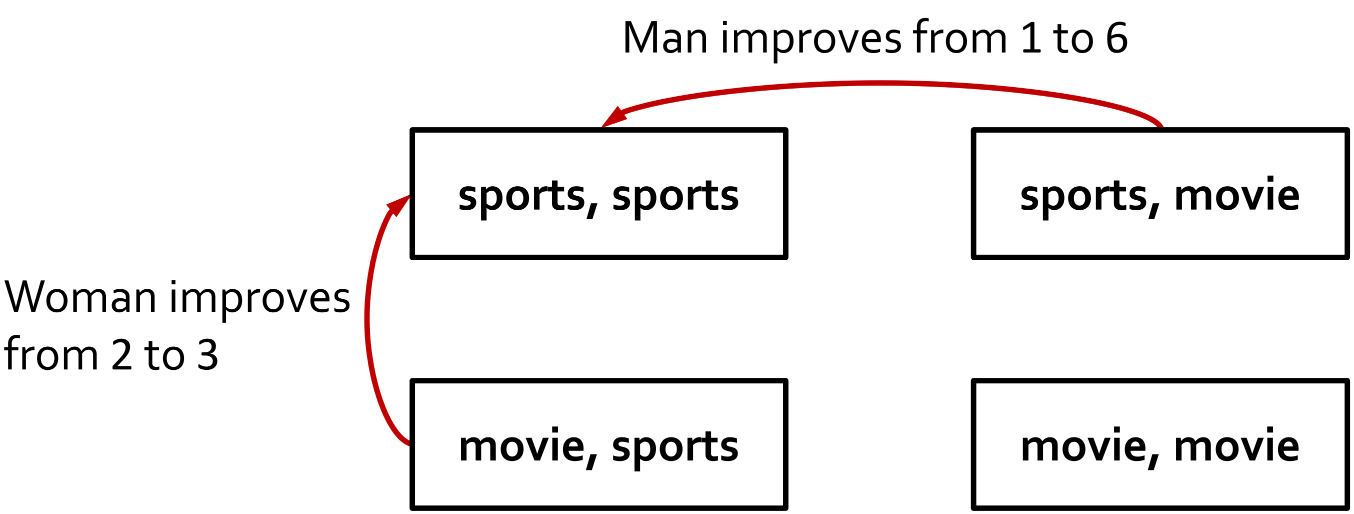

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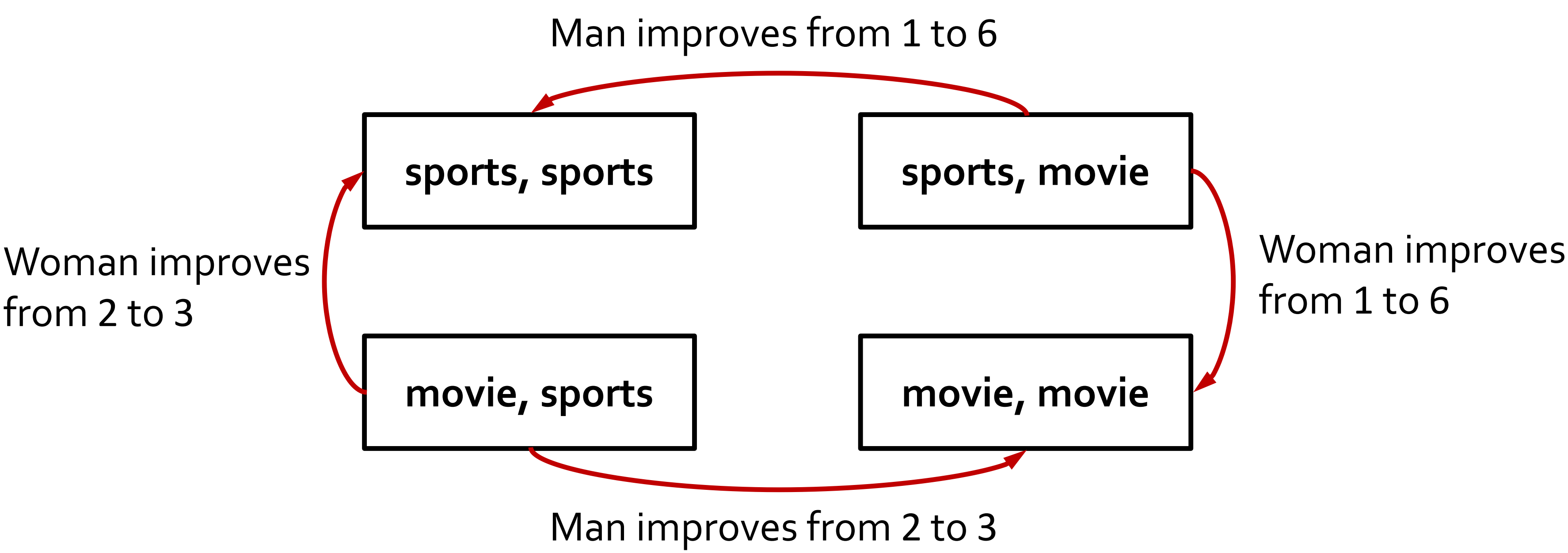

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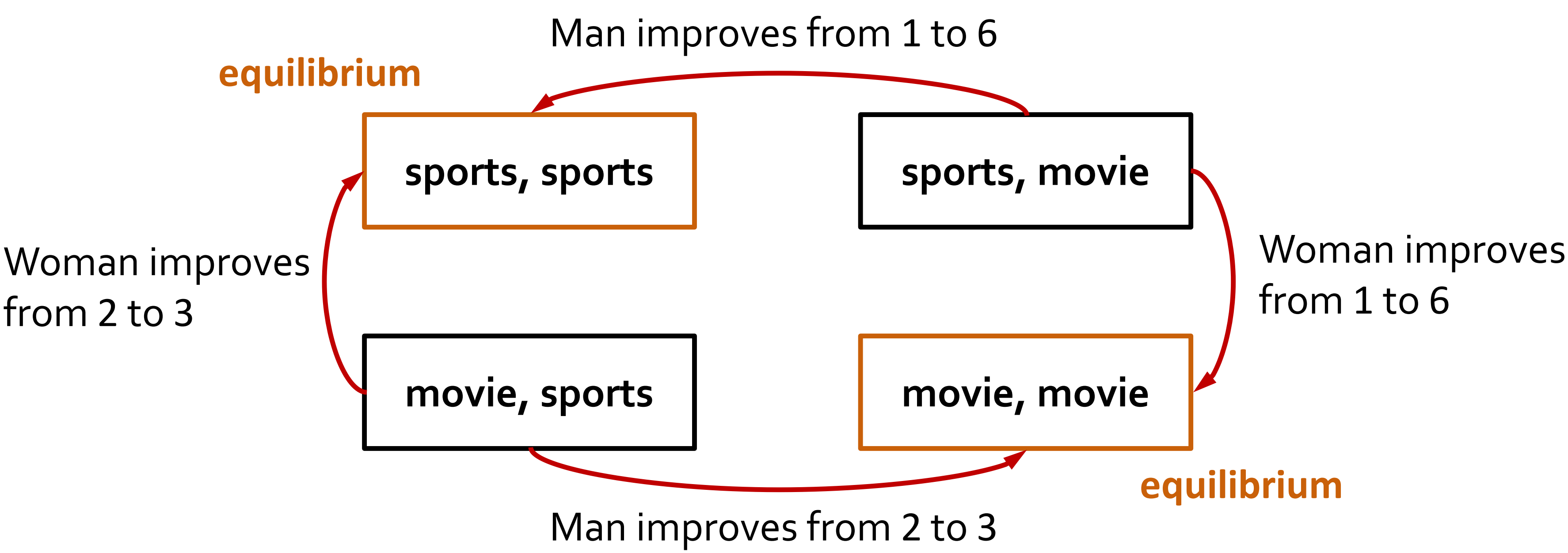

# Chicken

| | | column-driver | |
|---|---|---|---|
| | | **chicken** | **dare** |
| row-driver | **chicken** | **0, 0** | **-1, 1** |
| | **dare** | **1, -1** | **-10, -10** |

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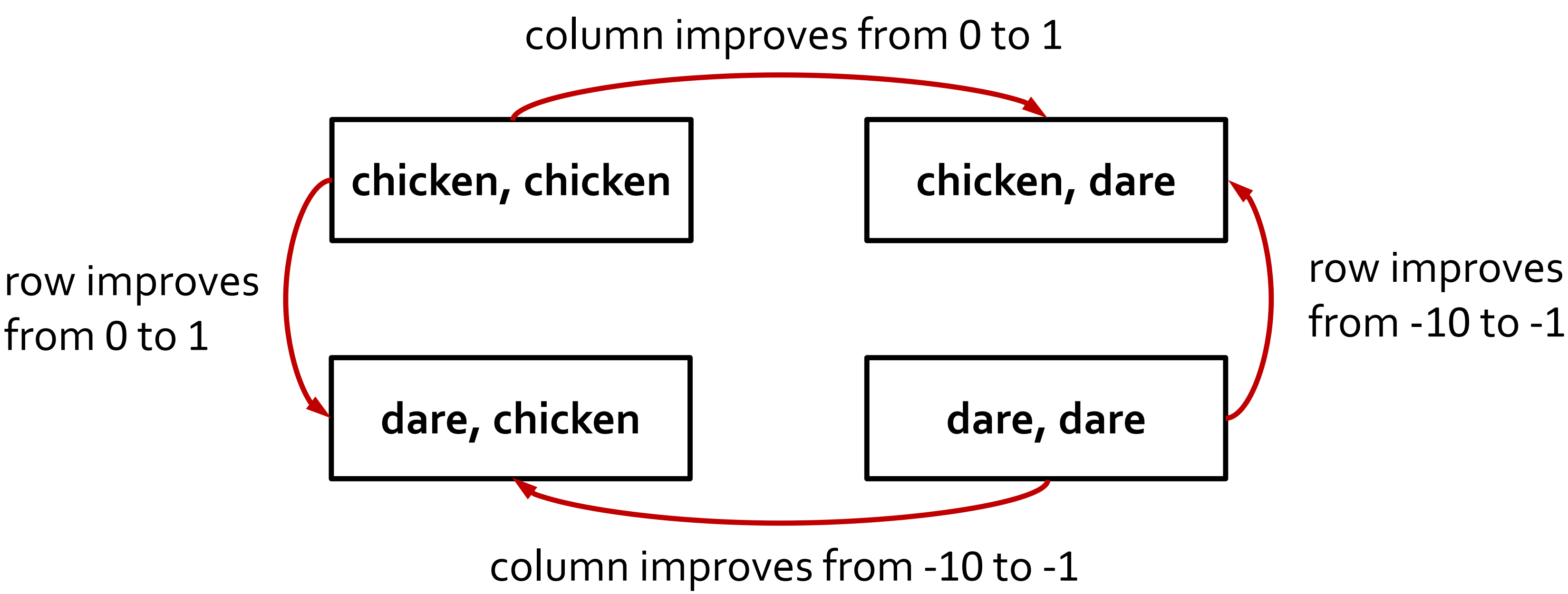

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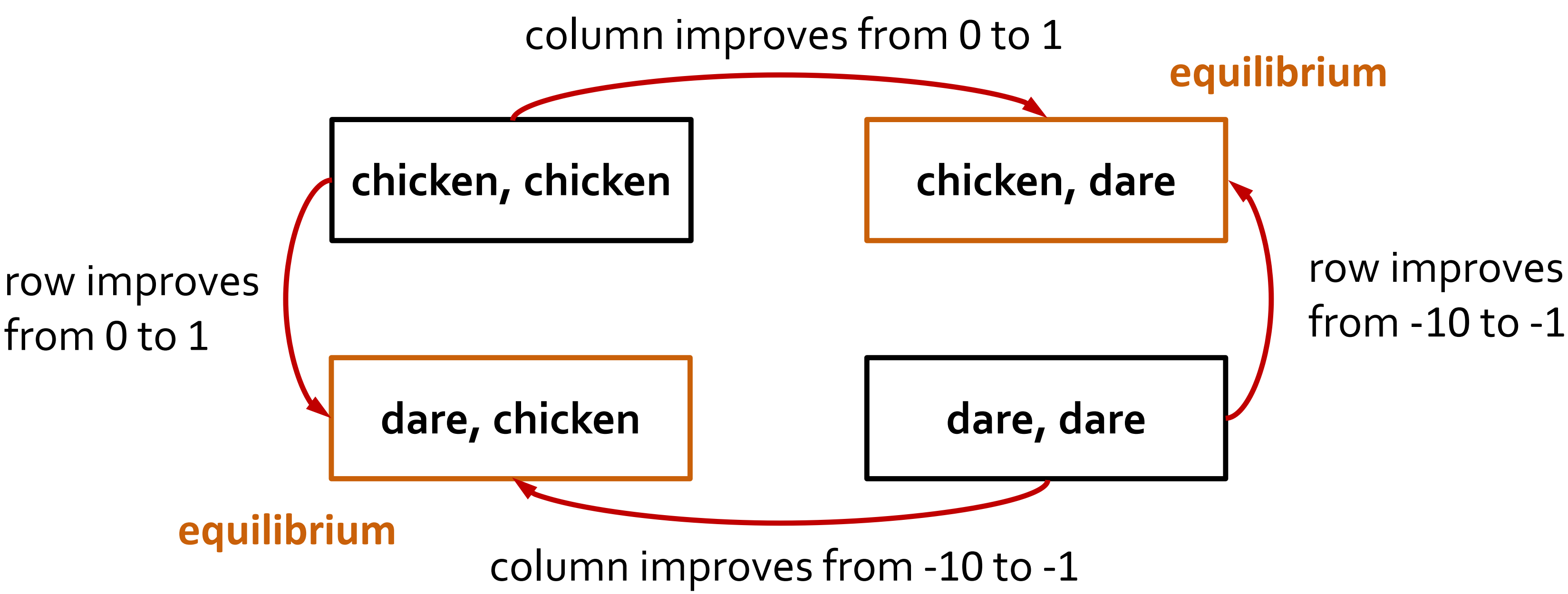
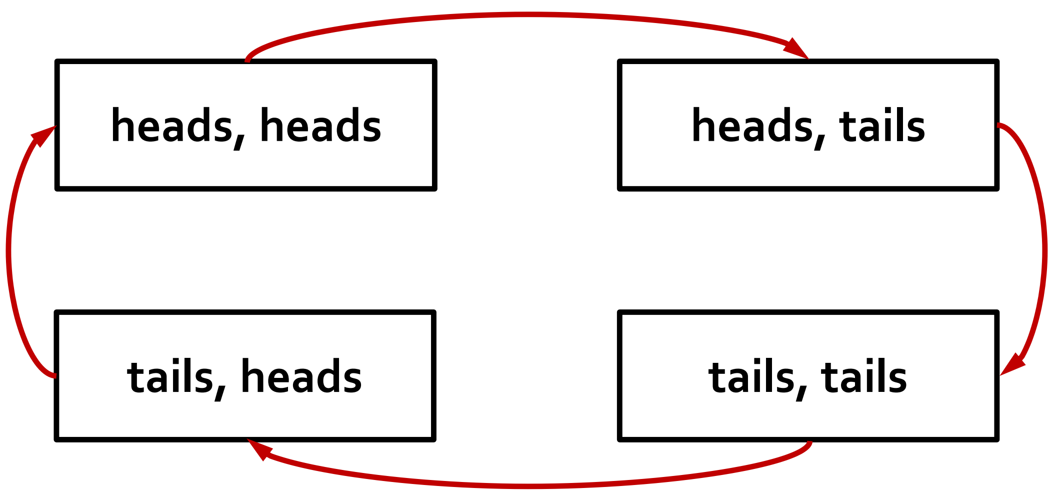


# Matching pennies

| even \ odd | heads | tails |
|---|---|---|
| **heads** | **1, -1** | **-1, 1** |
| **tails** | **-1, 1** | **1, -1** |

# Matching pennies

| | | odd | |
|---|---|---|---|
| | | heads | tails |
| even | heads | 1, -1 | -1, 1 |
| | tails | -1, 1 | 1, -1 |



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- The game is at a state $\boldsymbol{s} = (s_1, s_2, \dots, s_n)$ with probability

$$p(\boldsymbol{s}) = p_1(s_1) \cdot p_2(s_2) \cdot \dots \cdot p_n(s_n) = \prod_i p_i(s_i)$$

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$$p(\boldsymbol{s}) = p_1(s_1) \cdot p_2(s_2) \cdot \dots \cdot p_n(s_n) = \prod_i p_i(s_i)$$

- The **expected utility** of player $i$ is then

$$\mathbb{E}_p[u_i] = \sum_{\boldsymbol{s}} p(\boldsymbol{s}) \cdot u_i(\boldsymbol{s})$$

# Matching pennies

| | | odd | |
|---|---|---|---|
| | | heads | tails |
| even | heads | 1, -1 | -1, 1 | 0.8 |
| | tails | -1, 1 | 1, -1 | 0.2 |
| | | 0.4 | 0.6 |

# Matching pennies

| | | odd | | |
|---|---|---|---|---|
| | | **heads** | **tails** | |
| even | **heads** | **1, -1** | **-1, 1** | **0.8** |
| | **tails** | **-1, 1** | **1, -1** | **0.2** |
| | | **0.4** | **0.6** | |

- $p(\text{heads, heads}) = 0.8 \cdot 0.4 = 0.32$
- $p(\text{heads, tails}) = 0.8 \cdot 0.6 = 0.48$
- $p(\text{tails, heads}) = 0.2 \cdot 0.4 = 0.08$
- $p(\text{tails, tails}) = 0.2 \cdot 0.6 = 0.12$

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- $p(\text{heads, heads}) = 0.8 \cdot 0.4 = 0.32$
- $p(\text{heads, tails}) = 0.8 \cdot 0.6 = 0.48$
- $p(\text{tails, heads}) = 0.2 \cdot 0.4 = 0.08$
- $p(\text{tails, tails}) = 0.2 \cdot 0.6 = 0.12$

- $\mathbb{E}_p[u_\text{e}] = 0.32 \cdot 1 + 0.48 \cdot (-1) + 0.08 \cdot (-1) + 0.12 \cdot 1 = -0.12$
- $\mathbb{E}_p[u_\text{o}] = 0.32 \cdot (-1) + 0.48 \cdot 1 + 0.08 \cdot 1 + 0.12 \cdot (-1) = 0.12$

# Mixed equilibria

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**Theorem** [Nash, 1951]

Every finite strategic game of $n$ players has at least one mixed equilibrium

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- **Mixed equilibrium:** A profile of *mixed* strategies such that each player maximizes its expected utility, given the strategies of the other players

**Theorem** [Nash, 1951]

Every finite strategic game of $n$ players has at least one mixed equilibrium

- Every pure equilibrium is also a mixed equilibrium
  - Every pure strategy can be seen as a probability distribution over all strategies that assigns probability 1 to this one pure strategy

# Matching Pennies: mixed equilibria

| | | odd | |
|---|---|---|---|
| | | **heads** | **tails** |
| even | **heads** | **1, -1** | **-1, 1** |
| | **tails** | **-1, 1** | **1, -1** |

- Even player selects heads with probability $x$ and tails with $1 - x$
- Odd player selects heads with probability $y$ and tails with $1 - y$

# Matching Pennies: mixed equilibria

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|---|---|---|---|
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- Even player selects heads with probability $x$ and tails with $1 - x$
- Odd player selects heads with probability $y$ and tails with $1 - y$

- $p(\text{heads, heads}) = xy$
- $p(\text{heads, tails}) = x(1 - y)$
- $p(\text{tails, heads}) = (1 - x)y$
- $p(\text{tails, tails}) = (1 - x)(1 - y)$

# Matching Pennies: mixed equilibria

| | | odd | | |
|---|---|---|---|---|
| | | **heads** | **tails** | |
| even | **heads** | **1, -1** | **-1, 1** | $x$ |
| | **tails** | **-1, 1** | **1, -1** | $1-x$ |
| | | $y$ | $1-y$ | |

- $\mathbb{E}_p[u_{\mathrm{e}}]$
  $= xy \cdot 1 + x(1-y) \cdot (-1) + (1-x)y \cdot (-1) + (1-x)(1-y) \cdot 1$

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| | | odd: heads | odd: tails | |
|---|---|---|---|---|
| even | heads | **1, -1** | **-1, 1** | $x$ |
| even | tails | **-1, 1** | **1, -1** | $1-x$ |
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- $\mathbb{E}_p[u_\mathrm{e}]$
  $= xy \cdot 1 + x(1-y) \cdot (-1) + (1-x)y \cdot (-1) + (1-x)(1-y) \cdot 1$
  $= 4xy - 2x \; - 2y + 1$
  $= \boldsymbol{x(4y-2) - 2y + 1}$

# Matching Pennies: mixed equilibria

| | | odd | | |
|---|---|---|---|---|
| | | **heads** | **tails** | |
| even | **heads** | **1, -1** | **-1, 1** | $x$ |
| | **tails** | **-1, 1** | **1, -1** | $1 - x$ |
| | | $y$ | $1 - y$ | |

- $\mathbb{E}_p[u_\text{e}]$
  $= xy \cdot 1 + x(1 - y) \cdot (-1) + (1 - x)y \cdot (-1) + (1 - x)(1 - y) \cdot 1$
  $= 4xy - 2x - 2y + 1$
  $= \boldsymbol{x(4y - 2) - 2y + 1}$

- $\mathbb{E}_p[u_\text{o}]$
  $= xy \cdot (-1) + x(1 - y) \cdot 1 + (1 - x)y \cdot 1 + (1 - x)(1 - y) \cdot (-1)$
  $= \boldsymbol{y(2 - 4x) + 2x - 1}$

# Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_{\mathrm{e}}] = x(4y - 2) - 2y + 1$
- $\mathbb{E}_p[u_{\mathrm{o}}] = y(2 - 4x) + 2x - 1$

- The expected utility of each player is a **linear function** in terms of her corresponding probability

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- To analyze how a player is going to act, we need to see whether the slope of the linear function is negative or positive

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- **Negative:** the function is decreasing and the player aims to set a small value for the probability

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- The expected utility of each player is a **linear function** in terms of her corresponding probability
- To analyze how a player is going to act, we need to see whether the slope of the linear function is negative or positive

- **Negative:** the function is decreasing and the player aims to set a small value for the probability
- **Positive:** the function is increasing and the players aims to set a high value for the probability

# Matching Pennies: mixed equilibria

- $\mathbb{E}_p[u_\mathrm{e}] = x(4y - 2) - 2y + 1$
- $\mathbb{E}_p[u_\mathrm{o}] = y(2 - 4x) + 2x - 1$

- **Even player:** the slope is $4y - 2$ and it depends on $y$, the probability with which the odd player selects heads

# Matching Pennies: mixed equilibria

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- Following the same reasoning for the odd player, we can see that it must also be $\boldsymbol{x = 1/2}$
- For these values of $x$ and $y$ both slopes are equal to $0$ and the linear functions are maximized
- The pair $(x, y) = (1/2, 1/2)$ corresponds to a mixed equilibrium, which is actually unique for this game

# Unbalanced coordination

- Two players with two possible strategies A and B
- If both players select A, they get one point
- If both of them select B, they get two points
- If the select different strategies, they get zero points

| | | col player | |
|---|---|---|---|
| | | **A** | **B** |
| row player | **A** | **1, 1** | **0, 0** |
| | **B** | **0, 0** | **2, 2** |

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- Easy to verify that (A, A) and (B, B) are pure equilibria
- Are there any other mixed equilibria?

# Unbalanced coordination

| | | col player | |
|---|---|---|---|
| | | A | B |
| row player | A | 1, 1 | 0, 0 |
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- row player selects A with probability $x$ and B with $1 - x$
- col player selects A with probability $y$ and B with $1 - y$

- $p(\text{A, A}) = xy$
- $p(\text{A, B}) = x(1 - y)$
- $p(\text{B, A}) = (1 - x)y$
- $p(\text{B, B}) = (1 - x)(1 - y)$

# Unbalanced coordination

| | | col player: A | col player: B | |
|---|---|---|---|---|
| row player | **A** | **1, 1** | **0, 0** | $x$ |
| | **B** | **0, 0** | **2, 2** | $1-x$ |
| | | $y$ | $1-y$ | |

- $\mathbb{E}_p[u_{\mathrm{r}}]$
  $= xy \cdot 1 + x(1-y) \cdot 0 + (1-x)y \cdot 0 + (1-x)(1-y) \cdot 2$
  $= \boldsymbol{x(3y-2)+2-2y}$

- $\mathbb{E}_p[u_{\mathrm{c}}]$
  $= xy \cdot 1 + x(1-y) \cdot 0 + (1-x)y \cdot 0 + (1-x)(1-y) \cdot 2$
  $= \boldsymbol{y(3x-2)+2-2y}$

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  ⇨ the slope $3y - 2$ of $\mathbb{E}_p[u_\mathsf{r}]$ is **negative**

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- $(x,y)=(0,0)$ is a mixed equilibrium
- We already knew that: it corresponds to the pure equilibrium (A, A)

# Unbalanced coordination

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- $(x,y) = (1,1)$ is a mixed equilibrium corresponding to the pure equilibrium (B, B)

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- For $y = 2/3$ the slope $3y-2$ of $\mathbb{E}_p[u_\mathrm{r}]$ is zero and $\mathbb{E}_p[u_\mathrm{r}]$ is maximized by any choice of $x$, including $x = 2/3$

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- For $x = 2/3$ the slope $3x - 2$ of $\mathbb{E}_p[u_\text{c}]$ is zero and $\mathbb{E}_p[u_\text{c}]$ is maximized by any choice of y, including $y = 2/3$

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- For $\boldsymbol{x < 2/3}$ and $\boldsymbol{x > 2/3}$ we will reach to the same conclusion

- It remains to see what is going on for $\boldsymbol{x = 2/3}$ and $\boldsymbol{y = 2/3}$

- For $y = 2/3$ the slope $3y - 2$ of $\mathbb{E}_p[u_\mathrm{r}]$ is zero and $\mathbb{E}_p[u_\mathrm{r}]$ is maximized by any choice of $x$, including $x = 2/3$
- For $x = 2/3$ the slope $3x - 2$ of $\mathbb{E}_p[u_\mathrm{c}]$ is zero and $\mathbb{E}_p[u_\mathrm{c}]$ is maximized by any choice of y, including $y = 2/3$

- $(x, y) = (2/3, 2/3)$ is a fully mixed equilibrium of the game

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