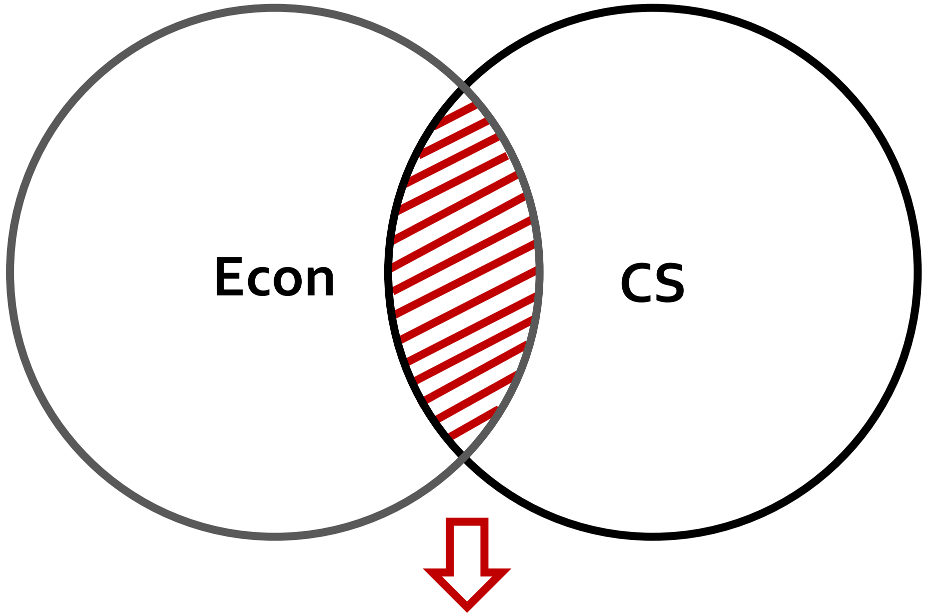
Design and analysis of algorithms for non-cooperative environments

Alexandros A. Voudouris

Department of Computer Engineering and Informatics
University of Patras



Design and analysis of algorithms for optimization problems, which deal with strategic agents, and require the use of *notions* and *tools* that have been developed in micro-economic theory (specifically, game theory)

Problems considered in this thesis

- The efficiency of resource allocation mechanisms for budget-constrained users
- Inefficiency in opinion formation games
- Mechanism design for ownership transfer
- Revenue maximization in take-it-or-leave-it sales

Resource allocation with budget constraints

Resource allocation

- One **divisible resource**
  - Bandwidth of a communication link
  - Processing time of a CPU
  - Storage space of a cloud

Resource allocation

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  - Bandwidth of a communication link
  - Processing time of a CPU
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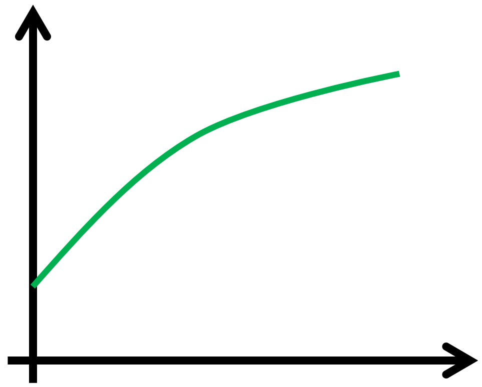
- n users such that user i has a **valuation function** $v_i: [0,1] \to \mathbb{R}_{\geq 0}$
  - $v_i(x)$ represents the value of user i for a fraction x of the resource
  - concave
  - non-decreasing
  - (semi-)differentiable



Resource allocation

Find an **allocation** $\boldsymbol{x} = (x_1, \ldots, x_n)$: $\sum_i x_i = 1$

to **maximize social welfare** $\mathrm{SW}(\boldsymbol{x}) = \sum_i v_i(x_i)$

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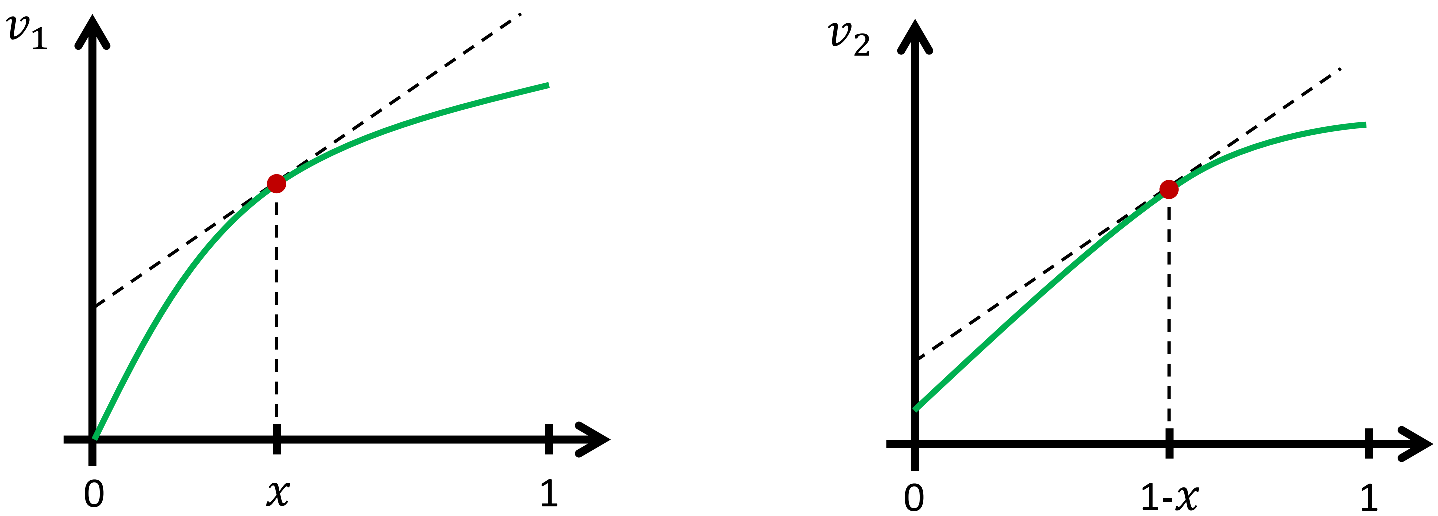
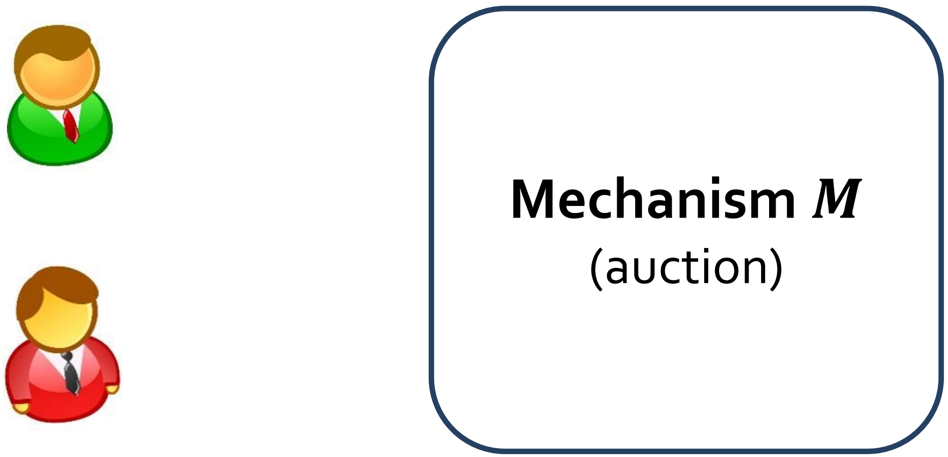


optimal allocation:

equal slopes

Resource allocation mechanisms



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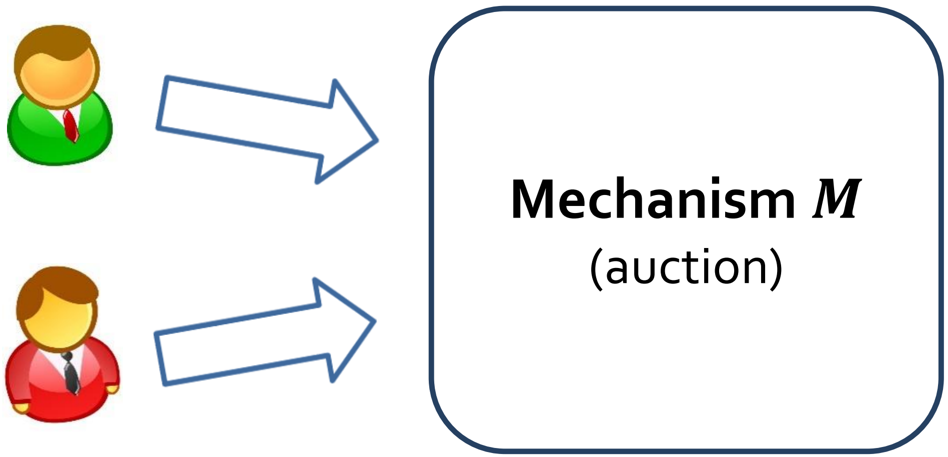
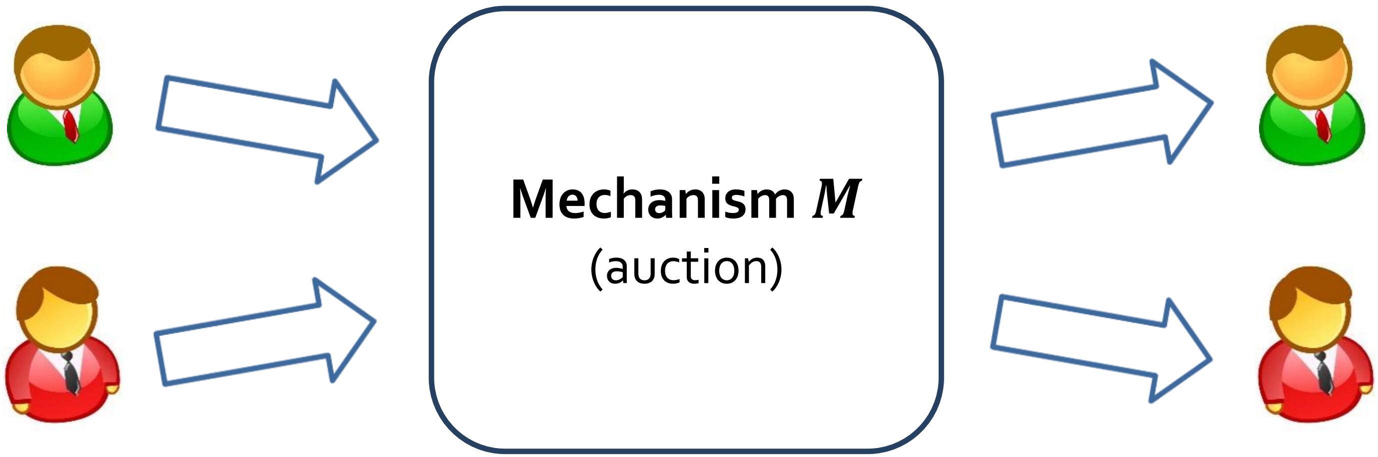


Input: signals (bids)

$$\boldsymbol{s} = (s_1, \ldots, s_n)$$

$$s_1, \ldots, s_n \geq 0$$

Resource allocation mechanisms



Input: signals (bids)

$$\boldsymbol{s} = (s_1, \dots, s_n)$$

$$s_1, \dots, s_n \geq 0$$

Output: allocation and payments

$$g(\boldsymbol{s}) = (g_1(\boldsymbol{s}), \dots, g_n(\boldsymbol{s}))$$

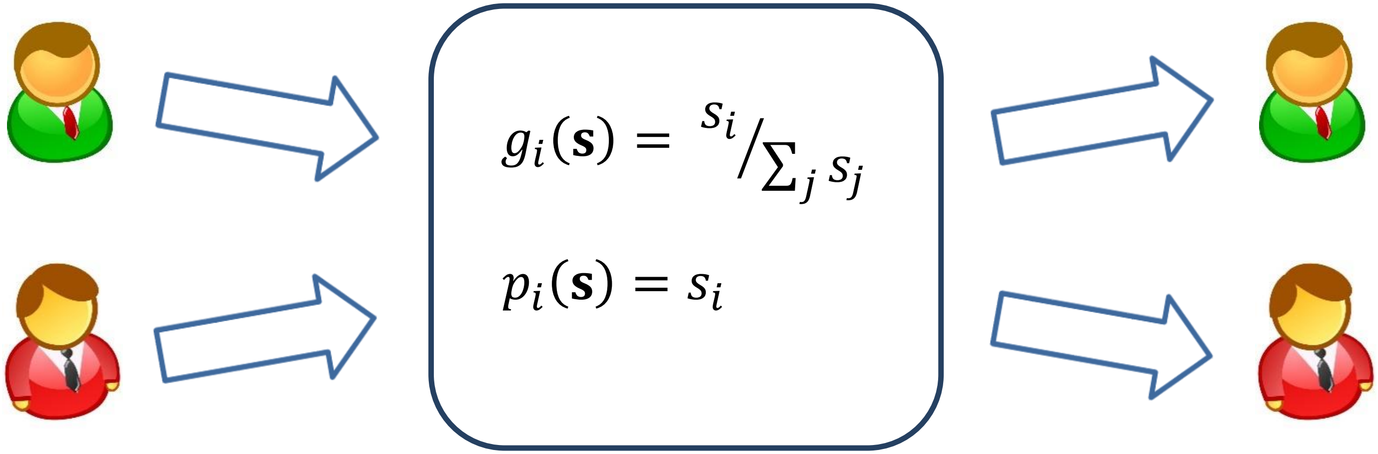
$$\sum_i g_i(\boldsymbol{s}) = 1$$

$$p(\boldsymbol{s}) = (p_1(\boldsymbol{s}), \dots, p_n(\boldsymbol{s}))$$

$$p_1(\boldsymbol{s}), \dots, p_n(\boldsymbol{s}) \geq 0$$

Examples

- **Kelly mechanism (1997)**
  - Proportional allocation
  - Pay-your-signal (PYS)



$$g_i(\mathbf{s}) = {}^{s_i}/_{\sum_j s_j}$$

$$p_i(\mathbf{s}) = s_i$$

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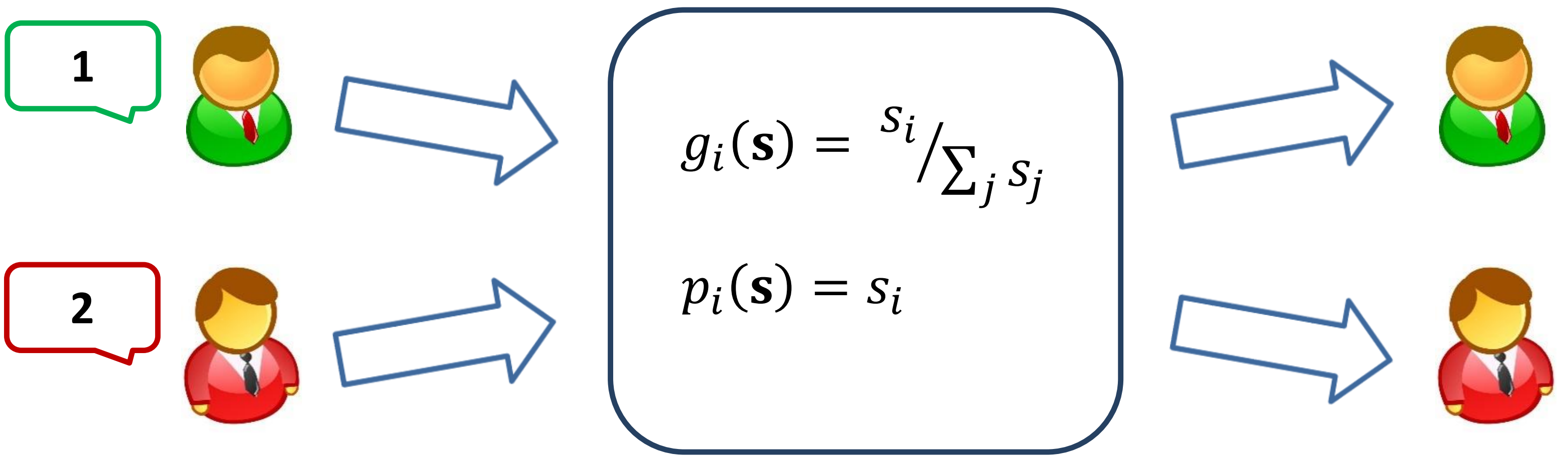

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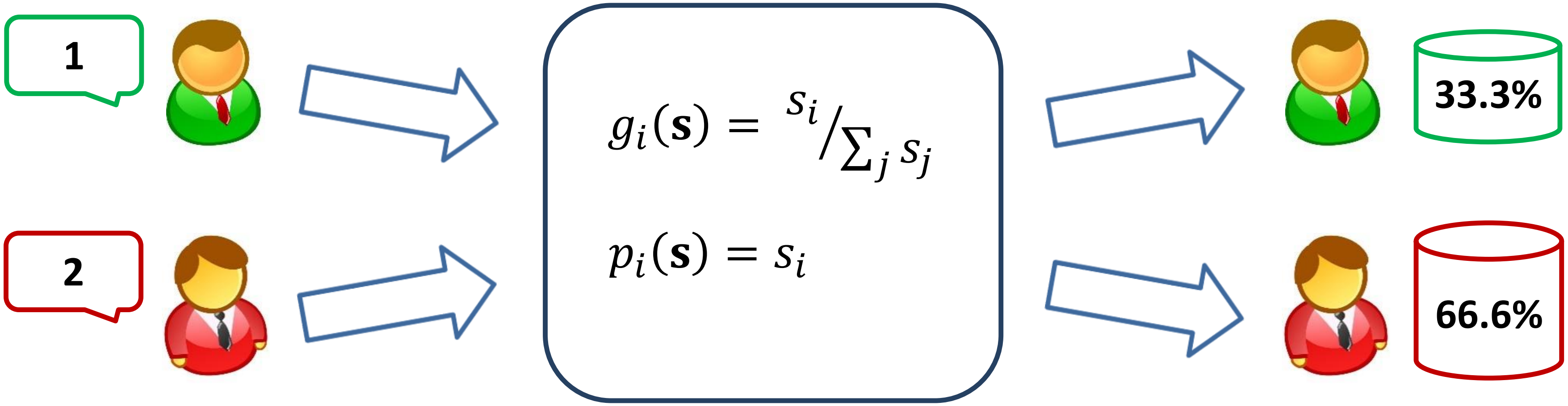

Examples

- **Sanghavi and Hajek (SH) mechanism (2004)**
  - Allocation depending on highest signal
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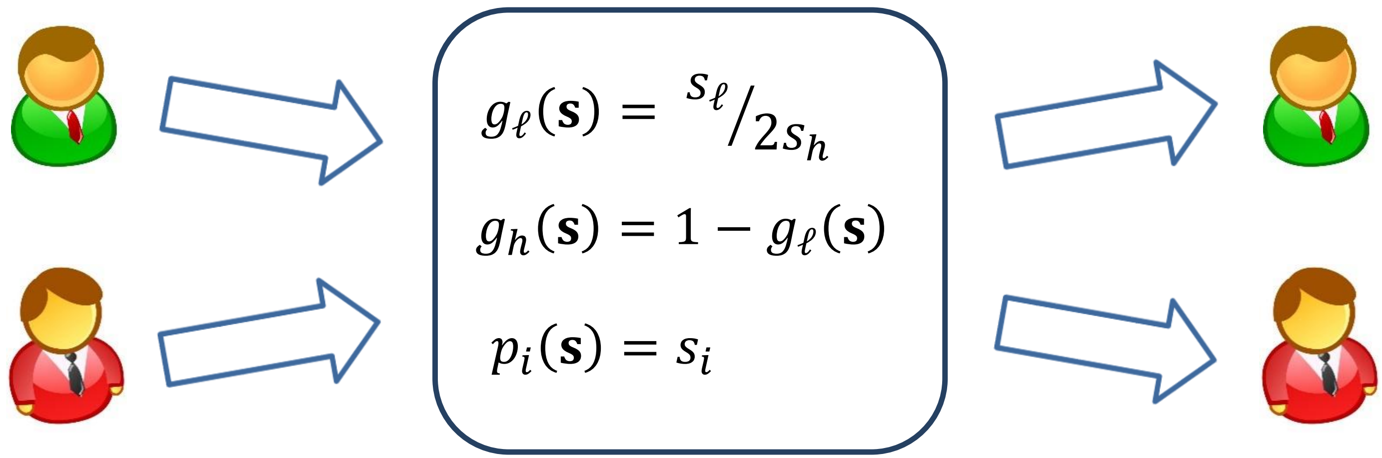

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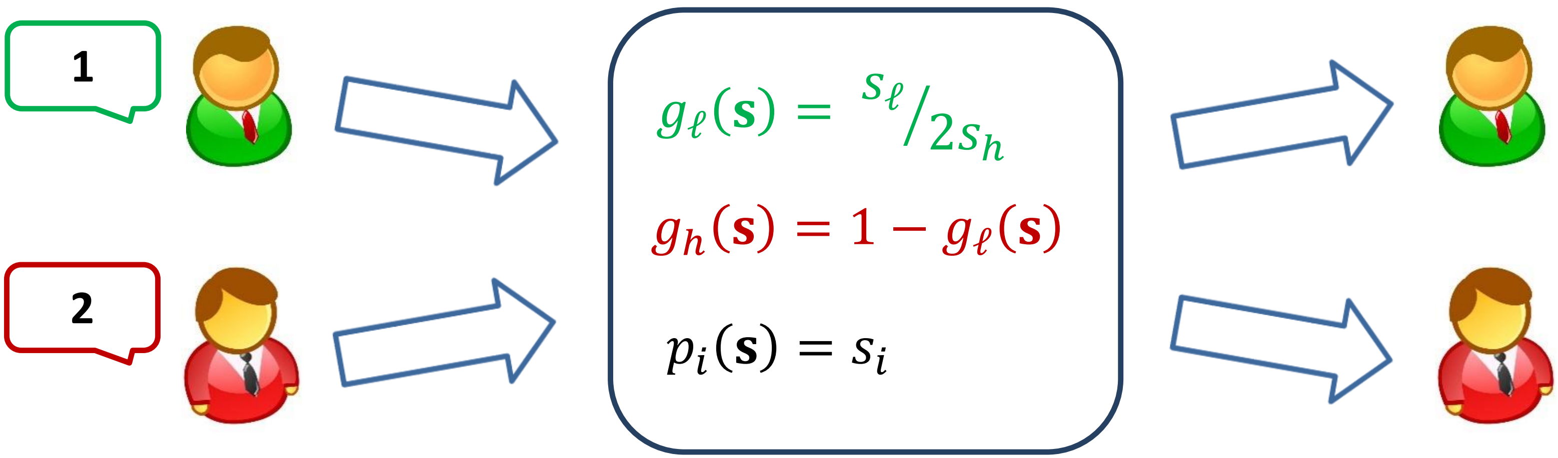

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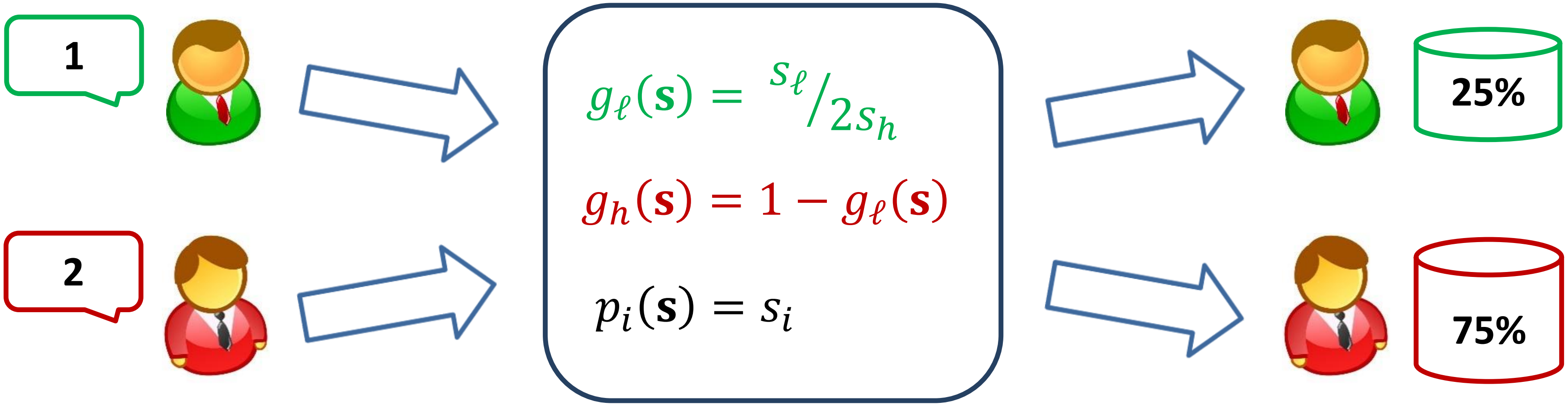

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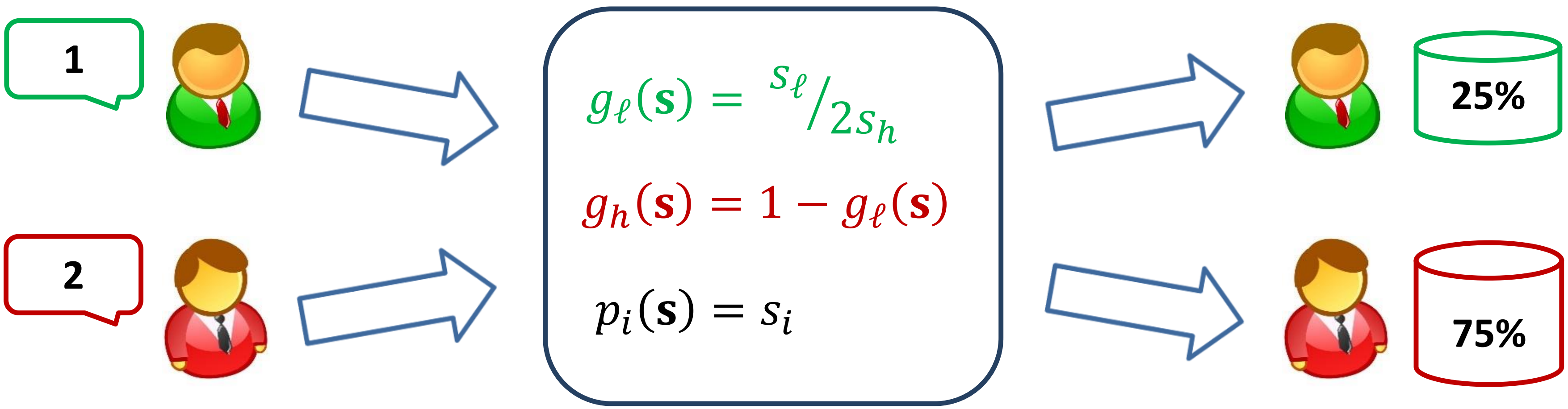
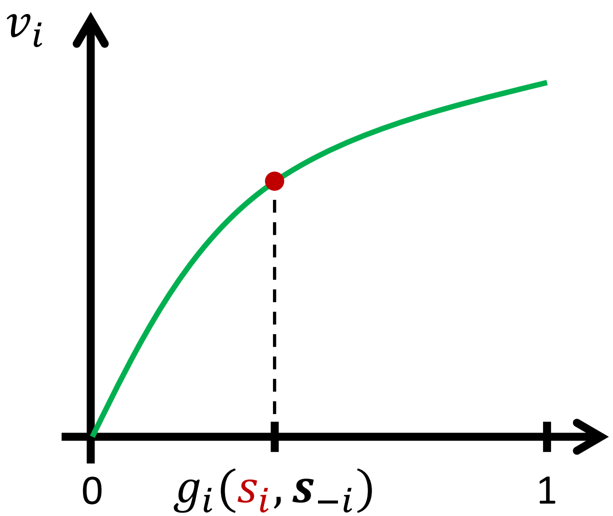


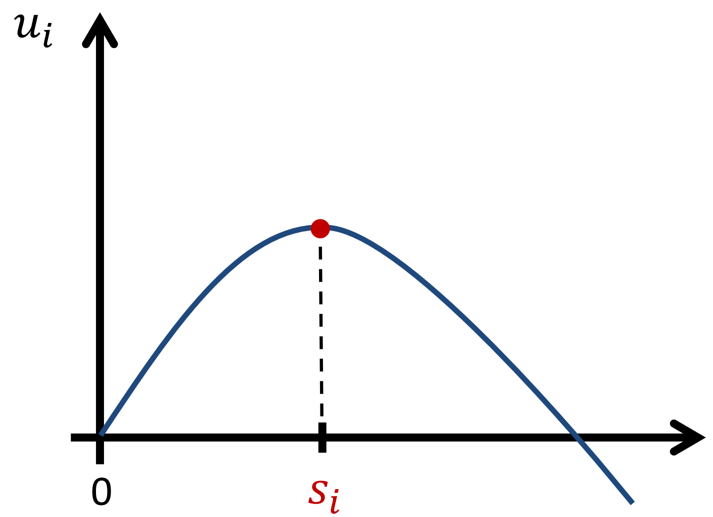
$$g_i(\mathbf{s}) = \frac{s_i}{\max_j s_j} \int_0^1 \prod_{k \neq i} \left(1 - \frac{s_k}{\max_j s_j} t\right) \mathrm{d}t$$

Strategic behavior

- Users are **utility-maximizers**

$$u_i(s_i, \mathbf{s}_{-i}) = \underbrace{v_i\big(g_i(s_i, \mathbf{s}_{-i})\big)}_{\text{value}} - \underbrace{p_i(s_i, \mathbf{s}_{-i})}_{\text{payment}}$$





Efficiency of mechanisms

- **(Pure Nash) equilibrium:** Given the signals of the other users, all users submit signals that maximize their personal utilities
- **Efficiency** of mechanism $\boldsymbol{M}$: **price of anarchy** with respect to the social welfare

$$\mathrm{PoA}(\boldsymbol{M}) = \sup_{\boldsymbol{v}} \frac{\max_{\boldsymbol{x}} \mathrm{SW}(\boldsymbol{x})}{\min_{\boldsymbol{s} \in \mathrm{EQ}(\boldsymbol{v}, \boldsymbol{M})} \mathrm{SW}(\boldsymbol{g}(\boldsymbol{s}))}$$

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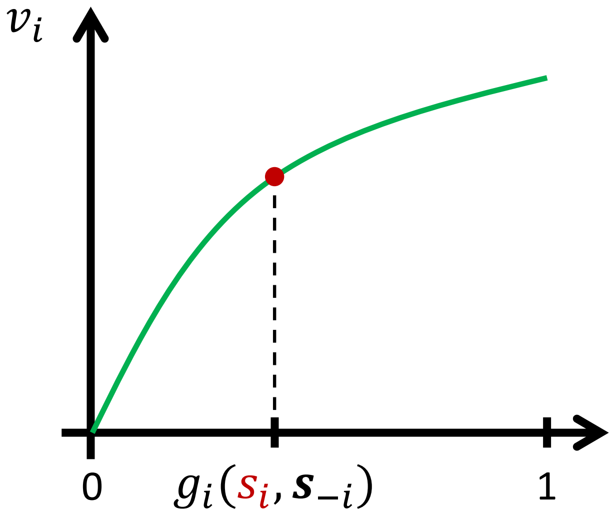
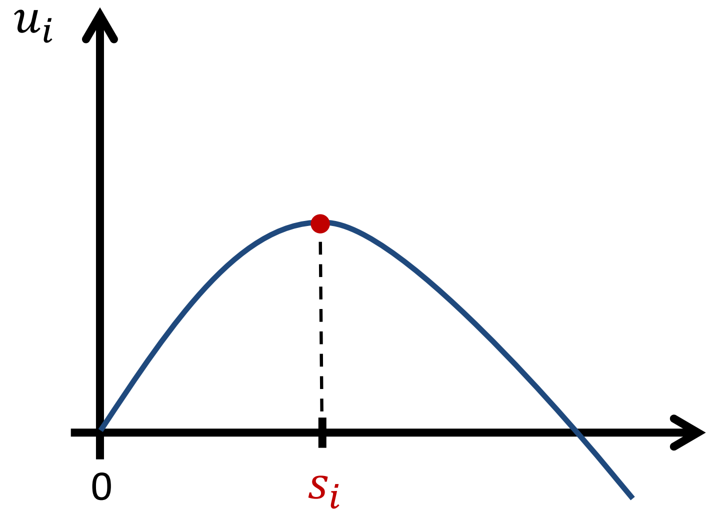
- **PoA(Kelly) = 4/3** (Johari & Tsitsiklis, 2004)
- **PoA(SH) = 8/7** (Sanghavi & Hajek, 2004)
- **There exist mechanisms with PoA = 1** (Maheswaran & Basar, 2006) (Yang & Hajek, 2007) (Johari & Tsitsiklis, 2009)

Worst-case characterization





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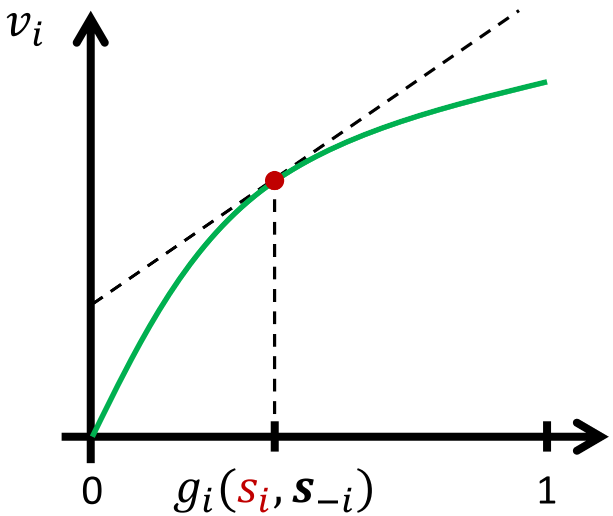

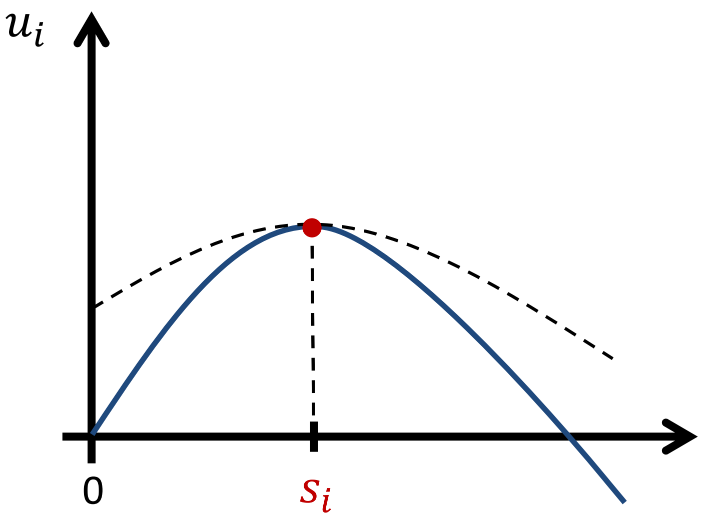

- The utility function that is defined by the **tangent** function is maximized at the same point

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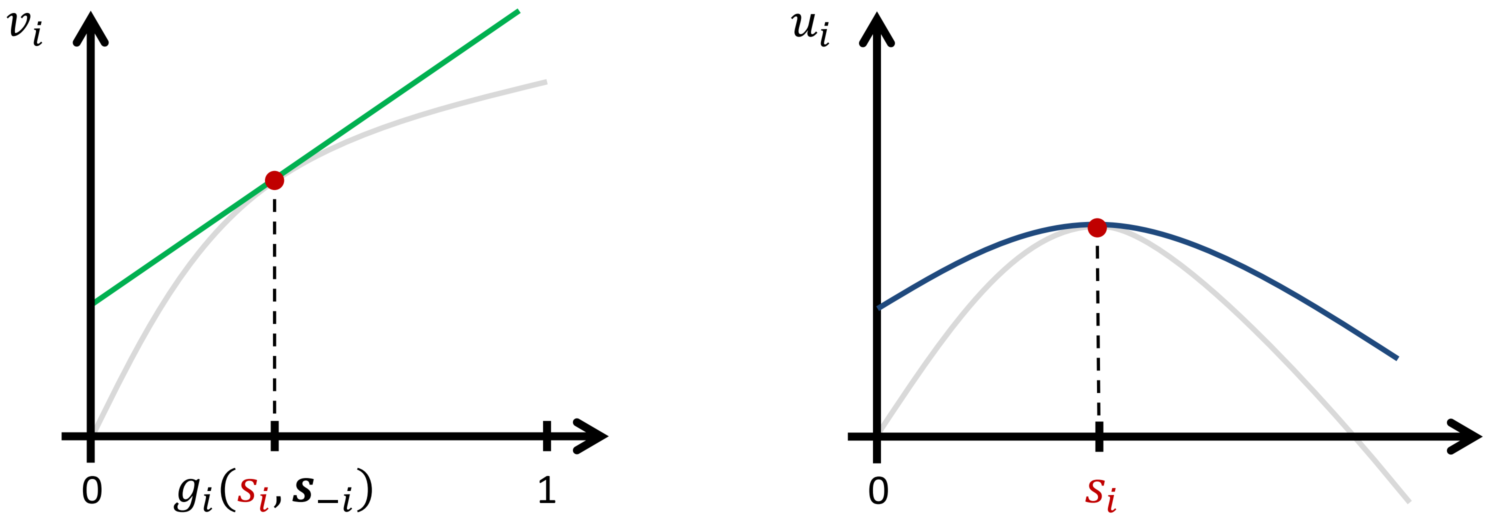

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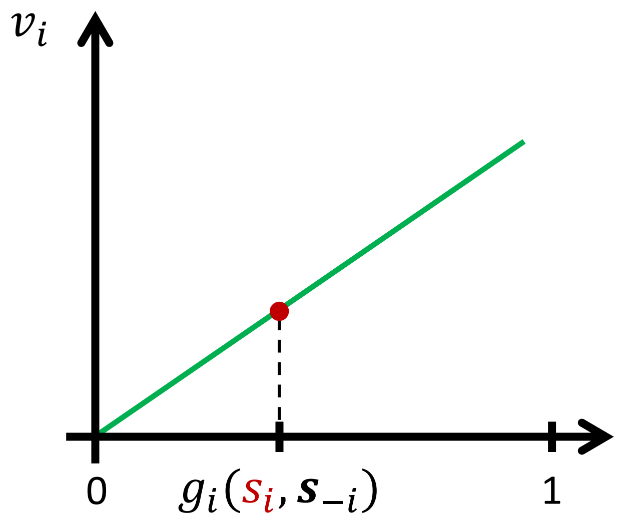
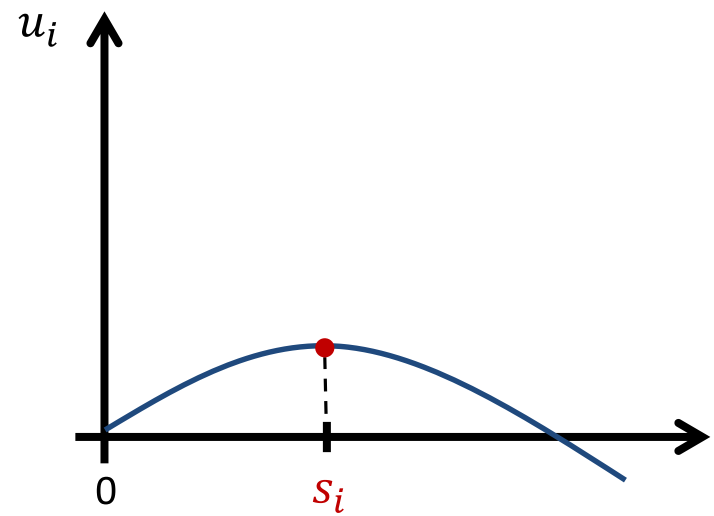




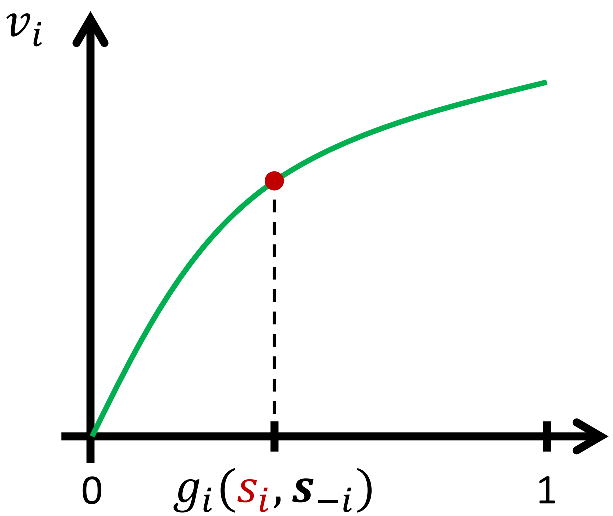
- The utility function that is defined by the **tangent** function is maximized at the same point
- The **same** signal vector would still be an equilibrium if the valuation functions were replaced by the tangents
- The price of anarchy can only become **worse**

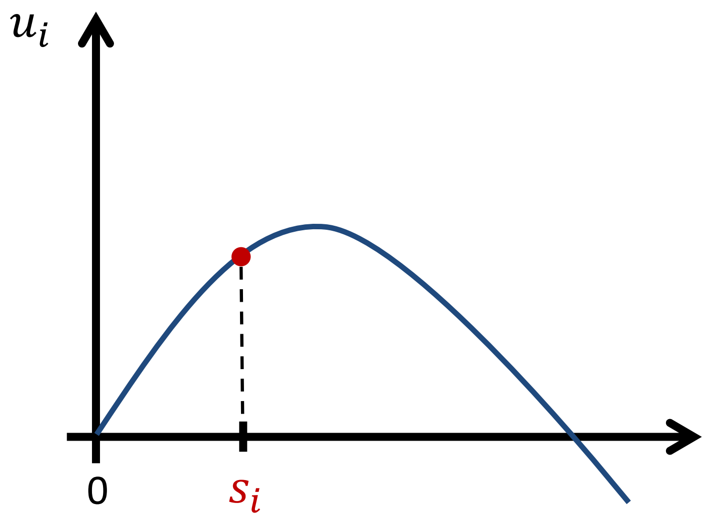
Budget constraints

- A more realistic model: each user has a **private budget** c_i which restricts the payments she can afford

Budget constraints

- A more realistic model: each user has a **private budget** c_i which restricts the payments she can afford
- The strategic behavior of every user is affected





- The game may reach to a **different equilibrium**

Efficiency under budget constraints

- The price of anarchy with respect to SW may be **arbitrarily bad**
  - high-value low-budget user *vs.* low-value high-budget user

Efficiency under budget constraints

- The price of anarchy with respect to SW may be **arbitrarily bad**
  - high-value low-budget user *vs.* low-value high-budget user
- **Liquid welfare**

$$\mathrm{LW}(\boldsymbol{x}) = \sum_i \min\{v_i(x_i), c_i\}$$

  - Syrgkanis and Tardos (2013)
  - Dobzinski and Paes Leme (2014)
- **Liquid price of anarchy:** price of anarchy with respect to the liquid welfare

$$\mathrm{LPoA}(\boldsymbol{M}) = \sup_{(\boldsymbol{v},\boldsymbol{c})} \frac{\max_{\boldsymbol{x}} \mathrm{LW}(\boldsymbol{x})}{\min_{\boldsymbol{s} \in \mathrm{EQ}((\boldsymbol{v},\boldsymbol{c}),\, \boldsymbol{M})} \mathrm{LW}(\boldsymbol{g}(\boldsymbol{s}))}$$

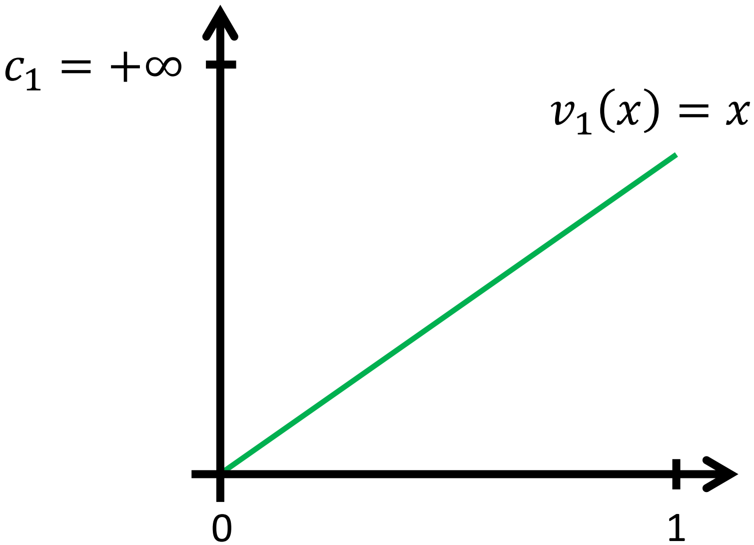
Lower bound for all mechanisms

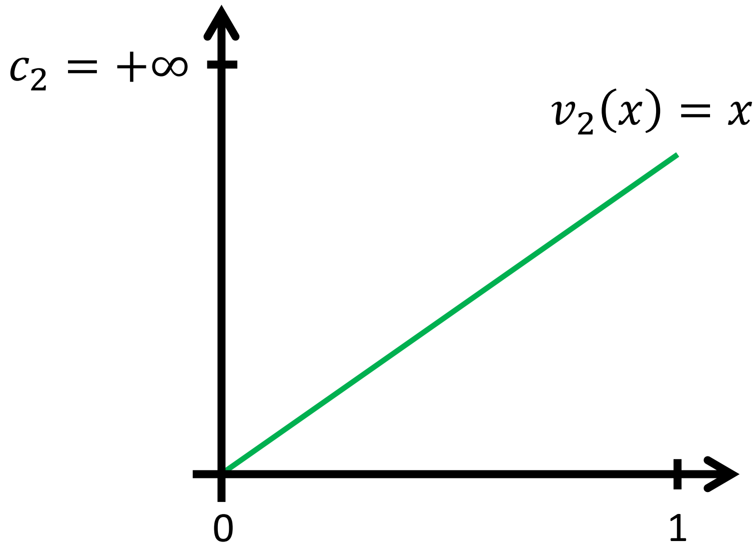
Theorem

Every resource allocation mechanism with n players has liquid price of anarchy at least $2 - 1/n$

Lower bound for all mechanisms

Theorem
Every resource allocation mechanism with 2 players has liquid price of anarchy at least 3/2





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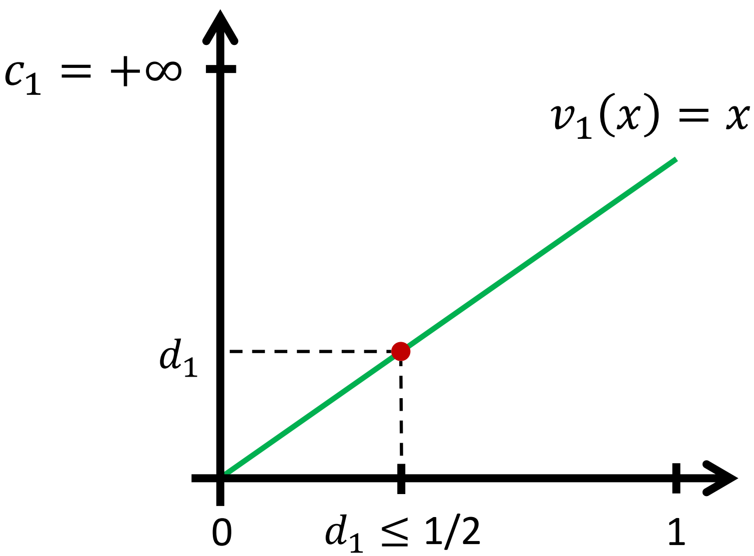

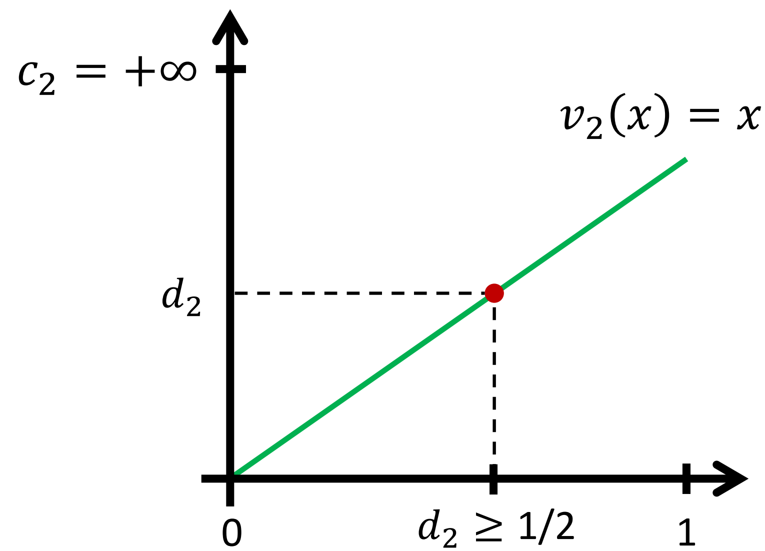

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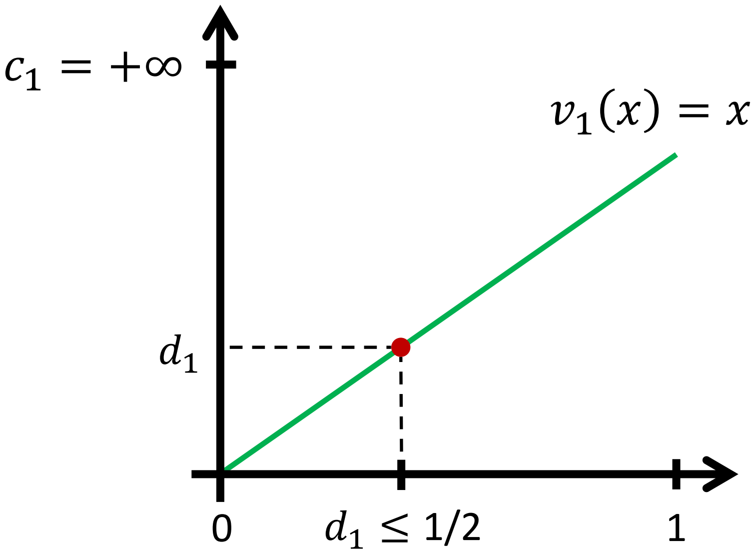

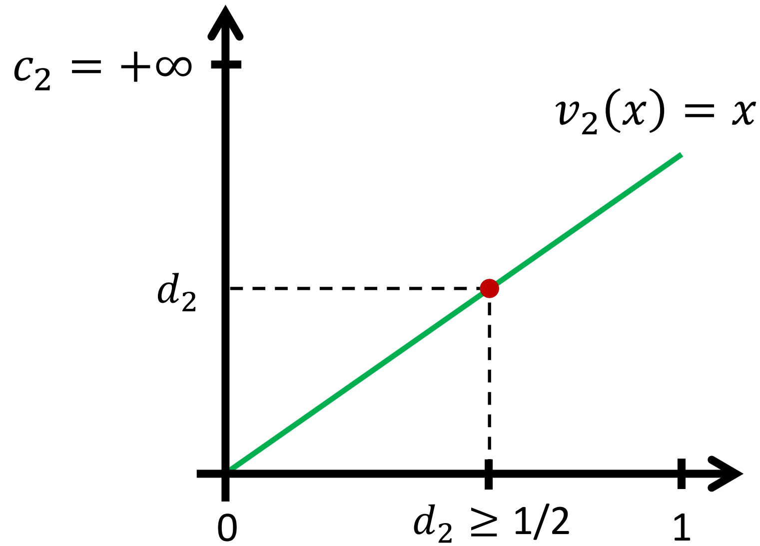

- The players have the same budget and valuation function
  $\Rightarrow$ liquid price of anarchy for this game $= 1$

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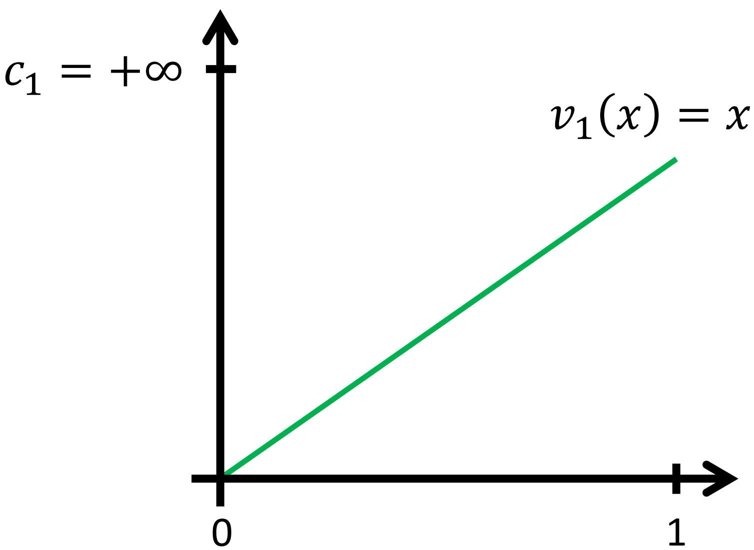

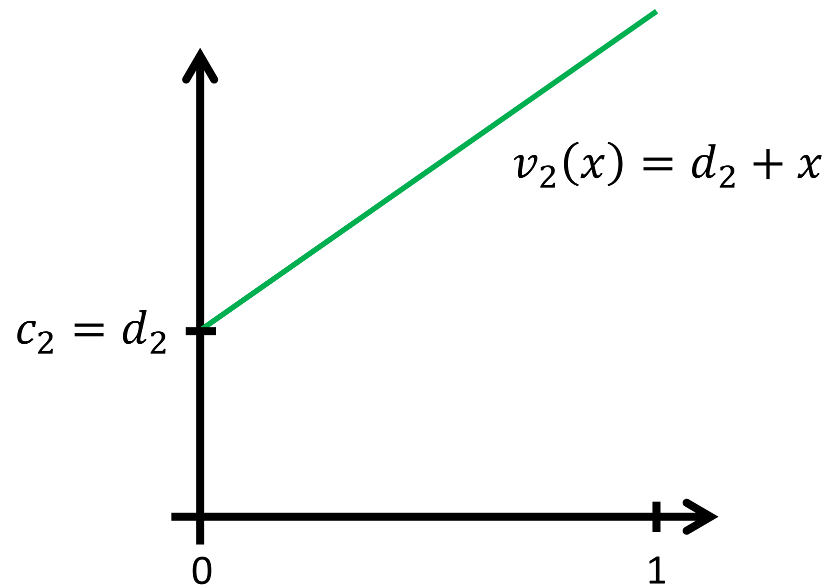

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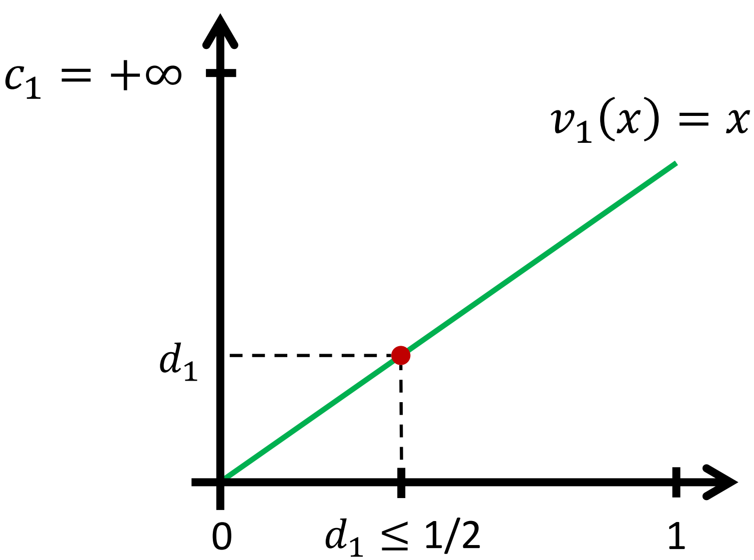

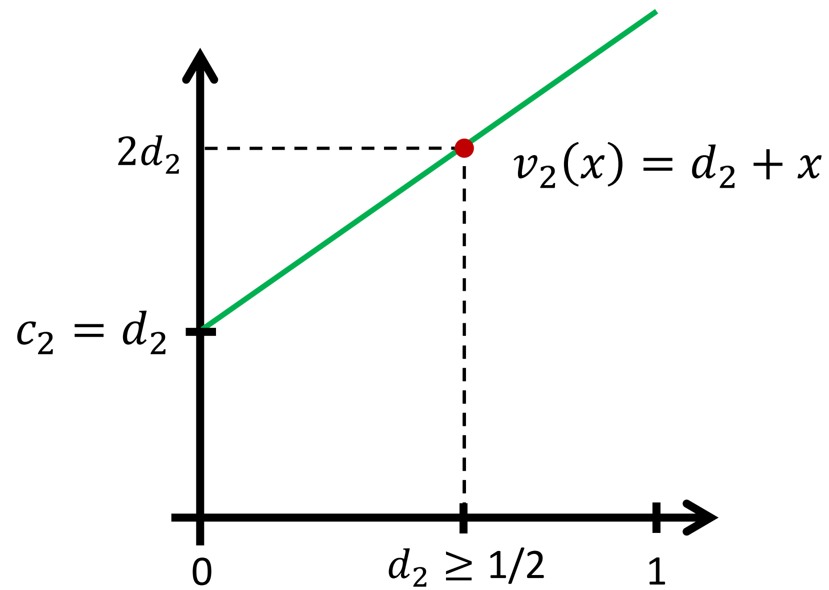

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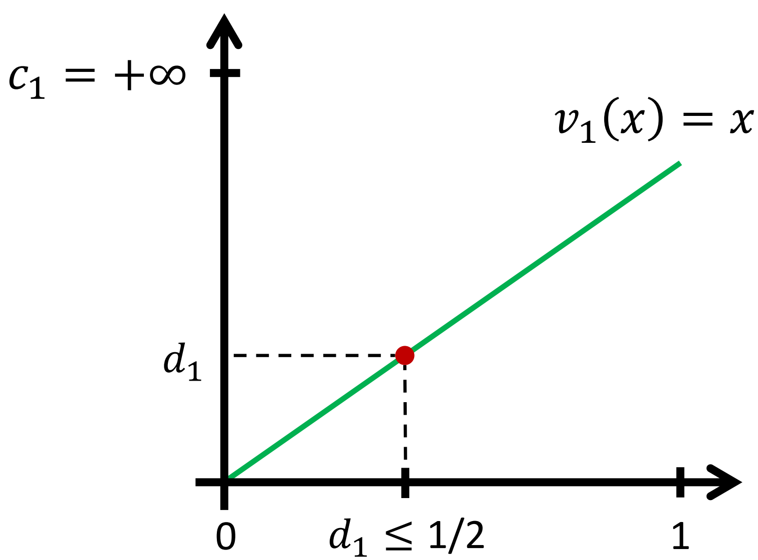

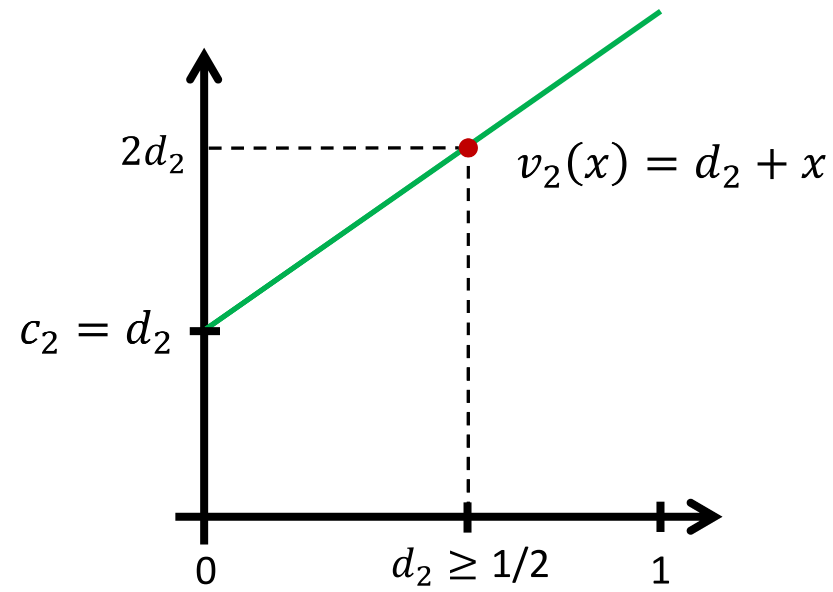

- Equilibrium: $\text{LW}(\boldsymbol{d}) = d_1 + d_2 = 1$

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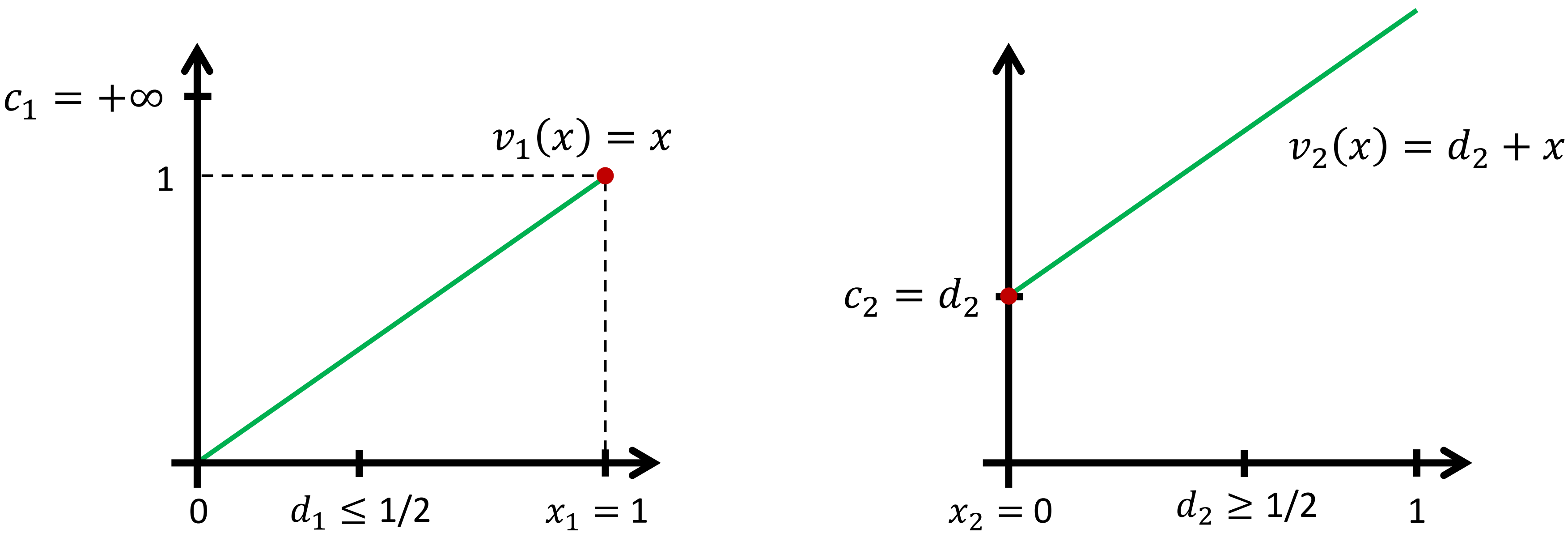

- Equilibrium: $\text{LW}(\boldsymbol{d}) = d_1 + d_2 = 1$
- Optimal allocation: $\text{LW}(\boldsymbol{x}) = 1 + d_2 \geq 3/2$ □

Worst-case characterization

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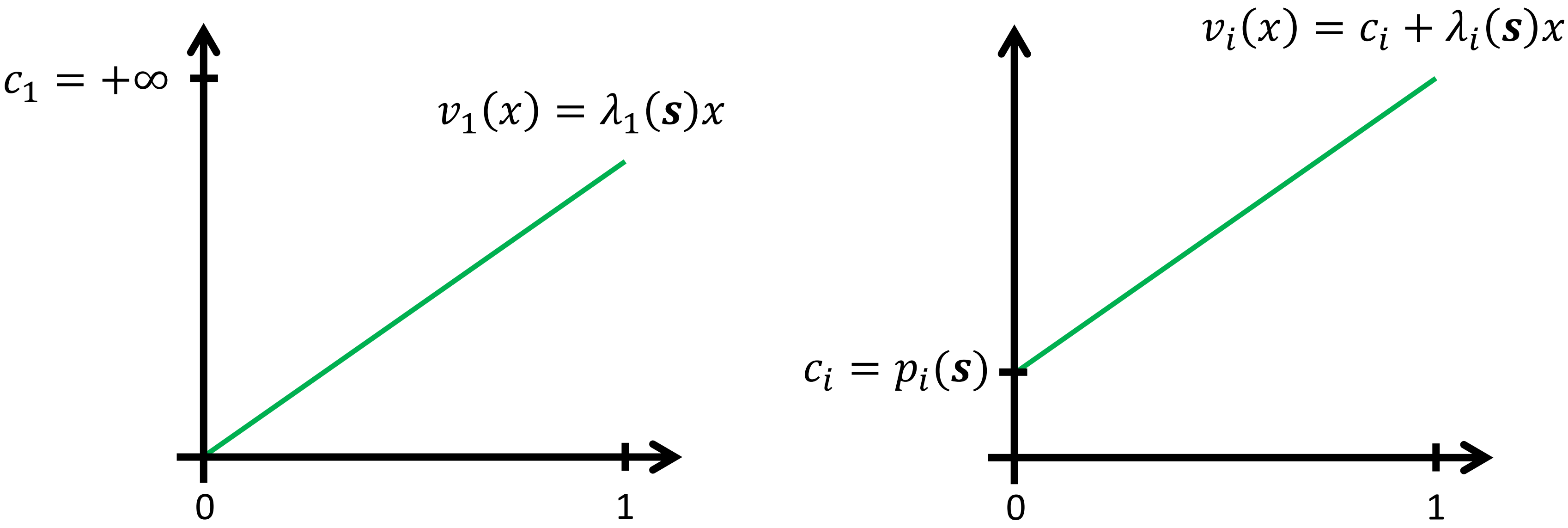
- For every $\boldsymbol{s}$, the worst case game where $\boldsymbol{s}$ is an equilibrium has a very special structure



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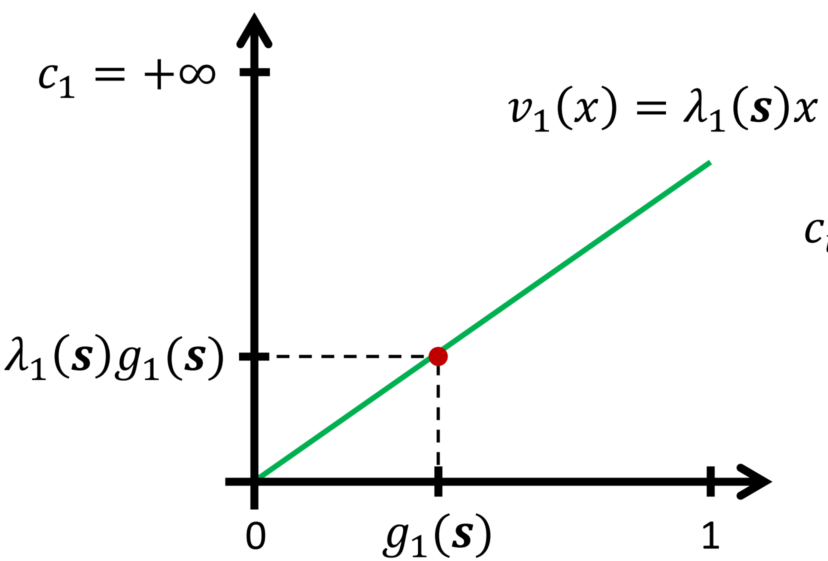

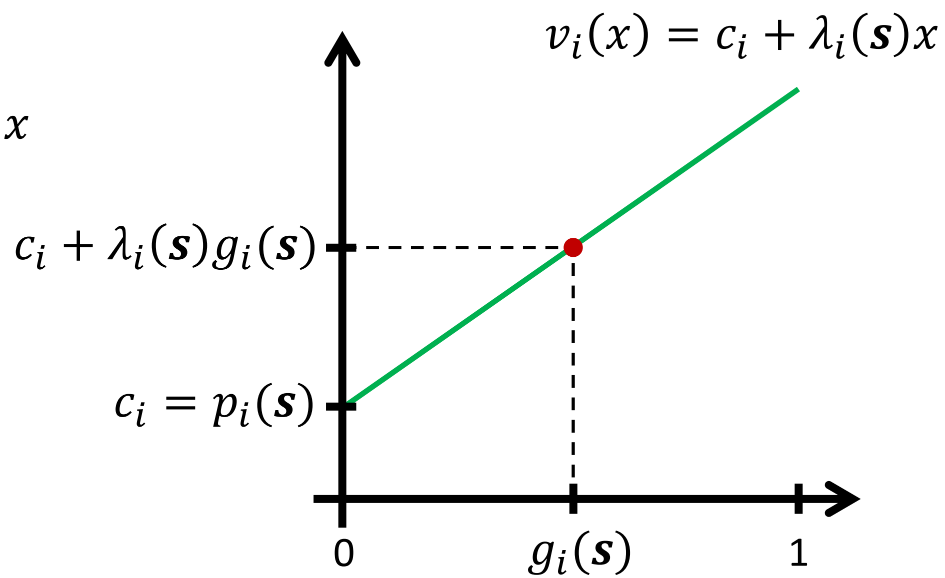

equilibrium

$$\text{LW}(g(\boldsymbol{s})) = \sum_{i\geq 2} p_i(s) + \lambda_1(\boldsymbol{s})g_1(\boldsymbol{s})$$

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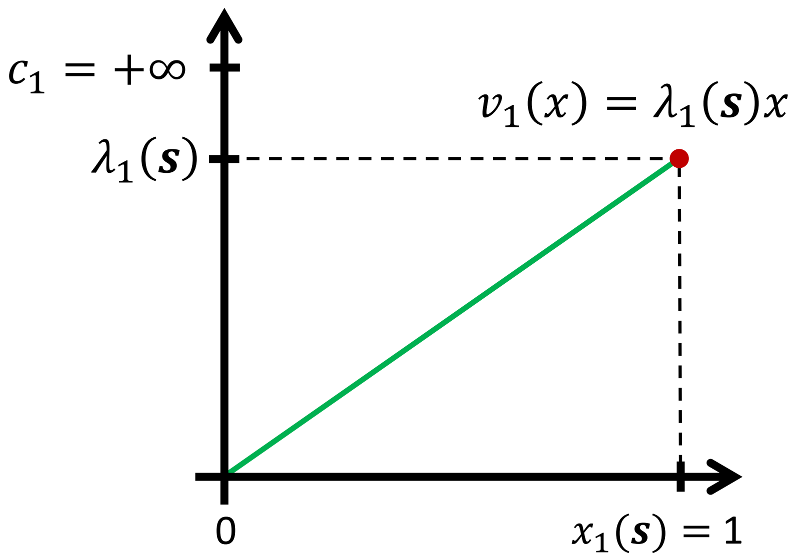

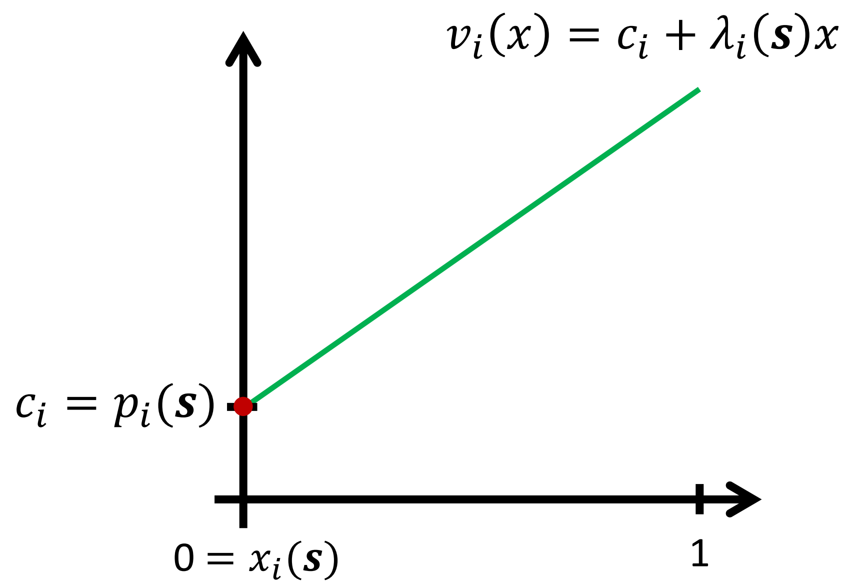

optimal allocation

$$\text{LW}(x(\boldsymbol{s})) = \sum_{i\geq 2} p_i(s) + \lambda_1(\boldsymbol{s})$$

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$$\text{LPoA}(\boldsymbol{s}\text{–game}) = \frac{\text{LW}(x(\boldsymbol{s}))}{\text{LW}(g(\boldsymbol{s}))} = \frac{\sum_{i \geq 2} p_i(\boldsymbol{s}) + \lambda_1(\boldsymbol{s})}{\sum_{i \geq 2} p_i(\boldsymbol{s}) + \lambda_1(\boldsymbol{s}) g_1(\boldsymbol{s})}$$

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<u>Theorem</u>

The liquid price of anarchy of mechanism $\boldsymbol{M}$ is

$$\text{LPoA}(\boldsymbol{M}) = \sup_{\boldsymbol{s}} \frac{\sum_{i\geq 2} p_i(\boldsymbol{s}) + \lambda_1(\boldsymbol{s})}{\sum_{i\geq 2} p_i(\boldsymbol{s}) + \lambda_1(\boldsymbol{s}) g_1(\boldsymbol{s})}$$

where:

$$\lambda_1(\boldsymbol{s}) = \left(\frac{\partial g_1(y, s_{-1})}{dy}\Big|_{y=s_1} \right)^{-1} \cdot \frac{\partial p_1(y, s_{-1})}{dy}\Big|_{y=s_1}$$

Tight bound for the Kelly mechanism

Theorem
The liquid price of anarchy of the Kelly mechanism is exactly 2

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The liquid price of anarchy of the Kelly mechanism is exactly 2

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$$\left.\begin{aligned} g_1(y, \boldsymbol{s}_{-1}) &= \frac{y}{y+C} \Rightarrow \frac{\partial g_1(y,\boldsymbol{s}_{-1})}{dy}\Big|_{y=s_1} = \frac{C}{(s_1+C)^2} \\ p_1(y, \boldsymbol{s}_{-1}) &= y \Rightarrow \frac{\partial p_1(y,\boldsymbol{s}_{-1})}{dy}\Big|_{y=s_1} = 1 \end{aligned}\right\} \lambda_1(\boldsymbol{s}) = \frac{(s_1+C)^2}{C}$$

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- Every player pays her signal: $\sum_{i\geq 2} p_i(\boldsymbol{s}) = \sum_{i\geq 2} s_i = C$
- For player 1: $g_1(\boldsymbol{s}) = \frac{s_1}{s_1+C}$

$$\left.\begin{aligned} g_1(y,\boldsymbol{s}_{-1}) = \frac{y}{y+C} &\Rightarrow \frac{\partial g_1(y,\boldsymbol{s}_{-1})}{dy}\Big|_{y=s_1} = \frac{C}{(s_1+C)^2} \\ p_1(y,\boldsymbol{s}_{-1}) = y &\Rightarrow \frac{\partial p_1(y,\boldsymbol{s}_{-1})}{dy}\Big|_{y=s_1} = 1 \end{aligned}\right\} \lambda_1(\boldsymbol{s}) = \frac{(s_1+C)^2}{C}$$

$$\text{LPoA}(\textbf{Kelly}) = \sup_{s_1,C} \frac{C + (s_1+C)^2/C}{C + s_1(s_1+C)/C} = 2$$

□

Overview of results

mechanism	LPoA
all	$\geq$ 2-1/n
Kelly	2
SH	3
E2-PYS	1.79
E2-SR	1.53

Overview of results

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all	$\geq 2\text{-}1/n$
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SH	3
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E2-SR	1.53

no mechanism can achieve full efficiency

Overview of results

mechanism	LPoA
all	$\geq$ 2-1/n
Kelly	2
SH	3
E2-PYS	1.79
E2-SR	1.53

almost best possible among all mechanisms with many players

Overview of results

mechanism	LPoA
all	$\geq$ 2-1/n
Kelly	2
SH	3
E2-PYS	1.79
E2-SR	1.53

different picture than the no-budget setting

Overview of results

mechanism	LPoA
all	$\geq$ 2-1/n
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E2-PYS	1.79
E2-SR	1.53

The allocation functions are solutions of simple ***linear differential equations***, which are defined by properly setting the payment function (PYS/SR) and using the worst-case characterization theorem

Overview of results

mechanism	LPoA
all	$\geq$ 2-1/n
Kelly	2
SH	3
E2-PYS	1.79
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best possible PYS mechanism
for two players

Overview of results

mechanism	LPoA
all	$\geq$ 2-1/n
Kelly	2
SH	3
E2-PYS	1.79
E2-SR	1.53

almost best possible mechanism
for two players

Opinion formation games

A simple model

- There is a set of individuals, and each of them has a (numerical) personal **belief** s_i
- However, she might express a possibly different **opinion** z_i
- **Averaging process:** all individuals simultaneously update their opinions according to the rule

$$z_i = \frac{s_i + \sum_{j \in N_i} z_j}{1 + |N_i|}$$

- N_i indicates the **social circle** of individual i
  - Friedkin & Johnsen (1990)

Game-theoretic interpretation

- The limit of the averaging process is the unique equilibrium of an **opinion formation game** that is defined by the personal beliefs of the individuals
- The opinions of the individuals (players) can be thought of as their **strategies**
- Each player has a **cost** that depends on her belief and the opinions that are expressed by other players in her social circle

$$\text{cost}_i(\boldsymbol{s}, \boldsymbol{z}) = (z_i - s_i)^2 + \sum_{j \in N_i} \left(z_i - z_j\right)^2$$

- The players act as **cost-minimizers**
  - Bindel, Kleinberg, & Oren (2015)

Co-evolutionary games

- The social circle of an individual changes as the opinions change
- ***k*-NN games** (Nearest Neighbors)
- There is **no** underlying **social network**
- The social circle $N_i(\boldsymbol{s}, \boldsymbol{z})$ consists of the k players with **opinions closest to** the belief of player i
- Same cost function

$$\text{cost}_i(\boldsymbol{s}, \boldsymbol{z}) = (z_i - s_i)^2 + \sum_{j \in N_i(\mathbf{s},\mathbf{z})} \left(z_i - z_j\right)^2$$

  – Bhawalkar, Gollapudi, & Munagala (2013)

Compromising opinion formation games

- **k-COF games**
- There is **no** underlying **social network**
- The social circle $N_i(\boldsymbol{s}, \boldsymbol{z})$ consists of the k players with **opinions closest to** the belief of player i
- Different cost function definition

$$\text{cost}_i(\boldsymbol{s}, \boldsymbol{z}) = \max_{j \in N_i(\mathbf{s}, \mathbf{z})} \{|z_i - s_i|, |z_i - z_j|\}$$

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  - Do pure equilibria always exist?
  - Can we efficiently compute them when they do exist?
  - How efficient are equilibria (price of anarchy and stability)?

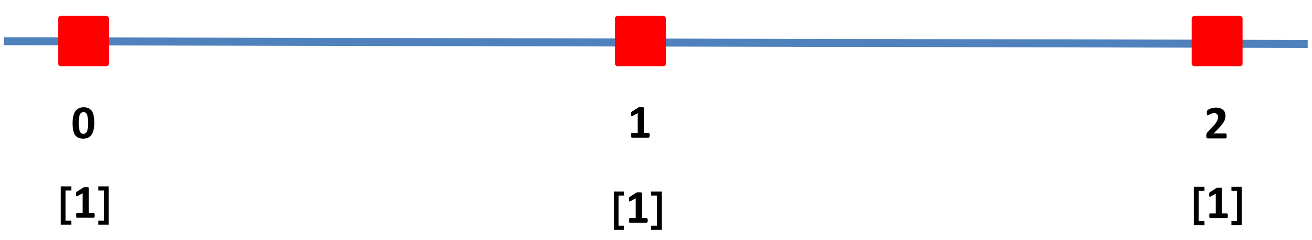
Existence of equilibria

Theorem

There exists a k-COF game with no pure equilibria, for any k

Existence of equilibria

<u>Theorem</u>
There exists a k-COF game with no pure equilibria, for $k = 1$



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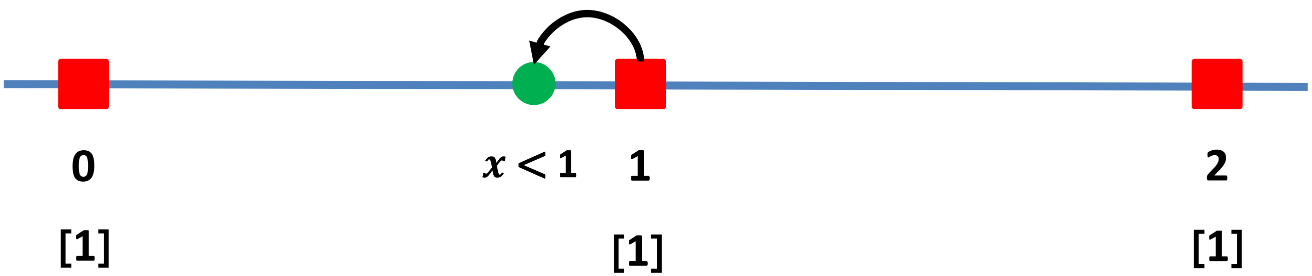

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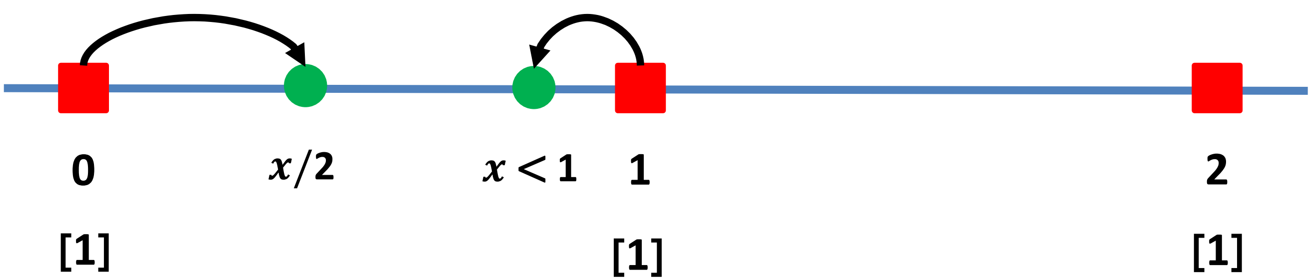

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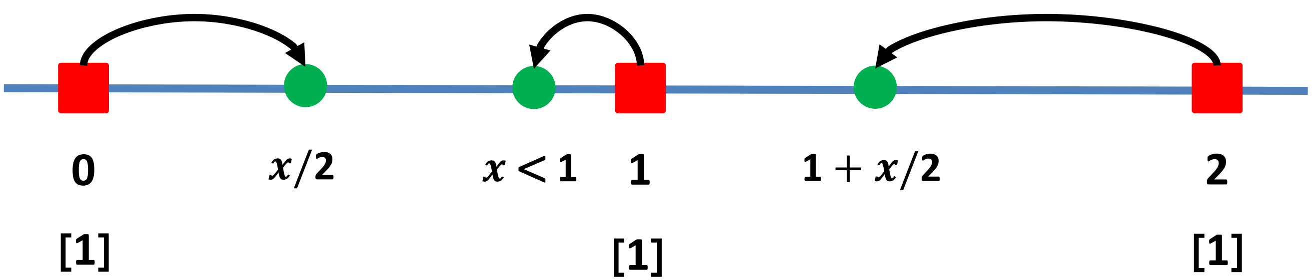

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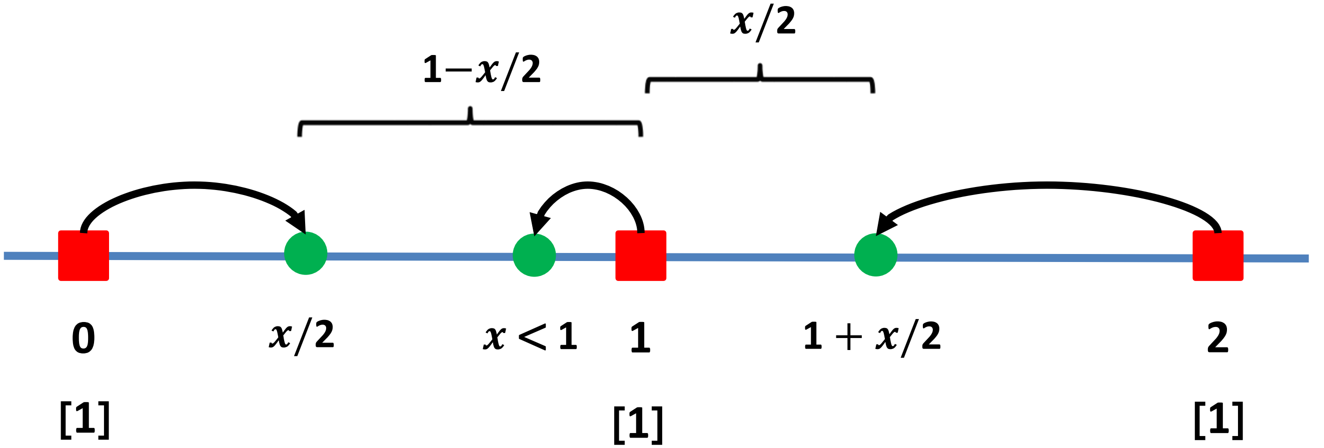

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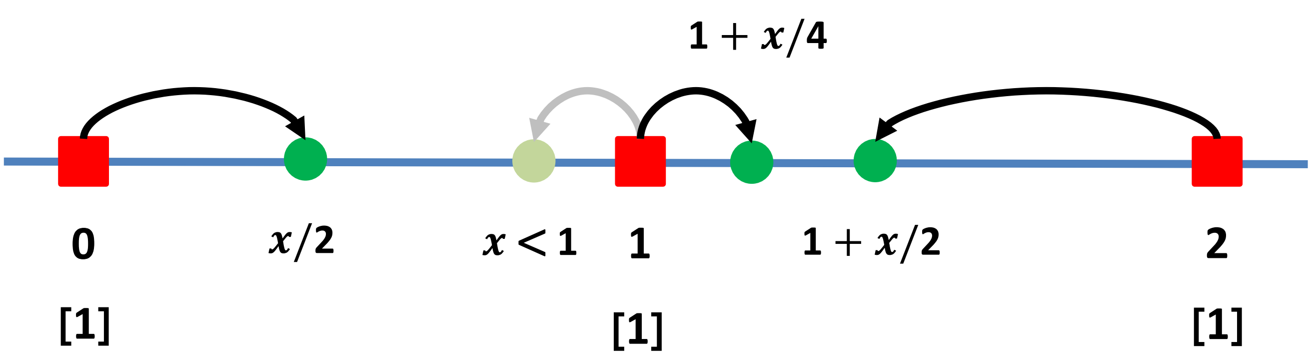

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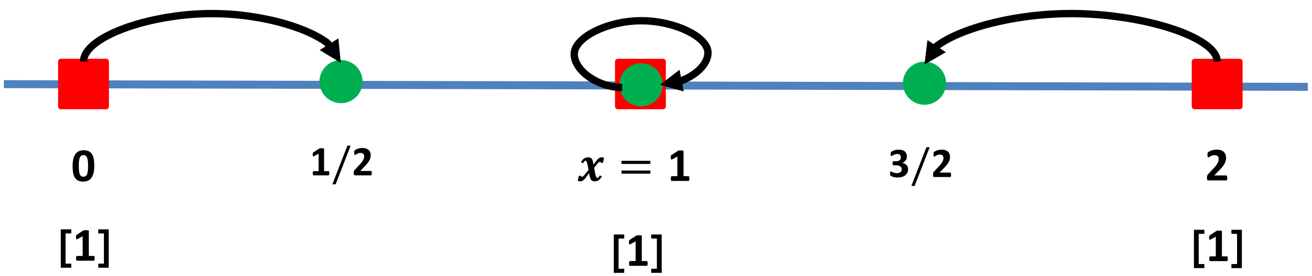

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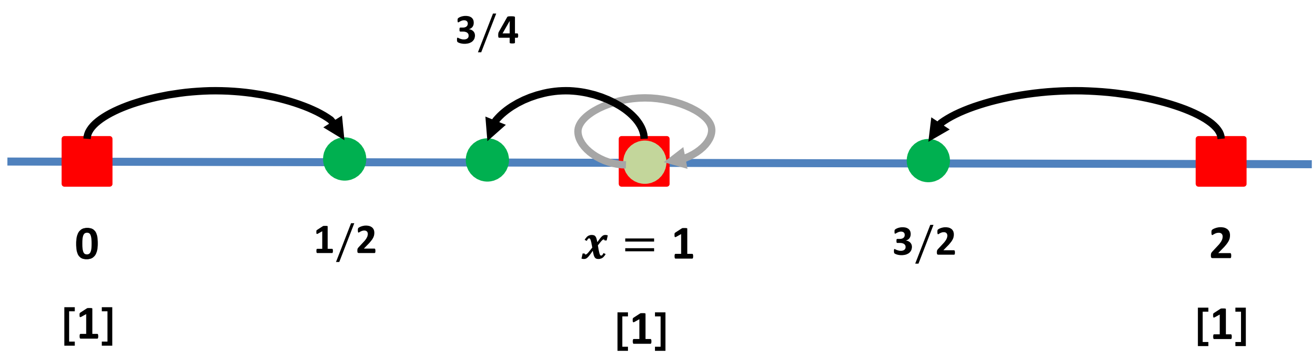

□

A lower bound on the price of anarchy

Theorem
For $k = 1$, the price of anarchy is at least 3

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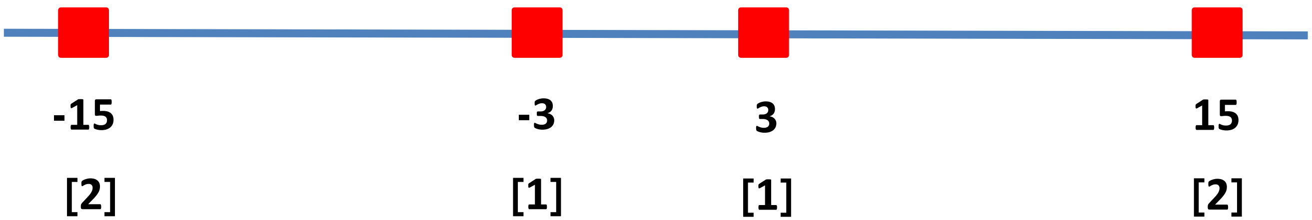

A lower bound on the price of anarchy

Theorem

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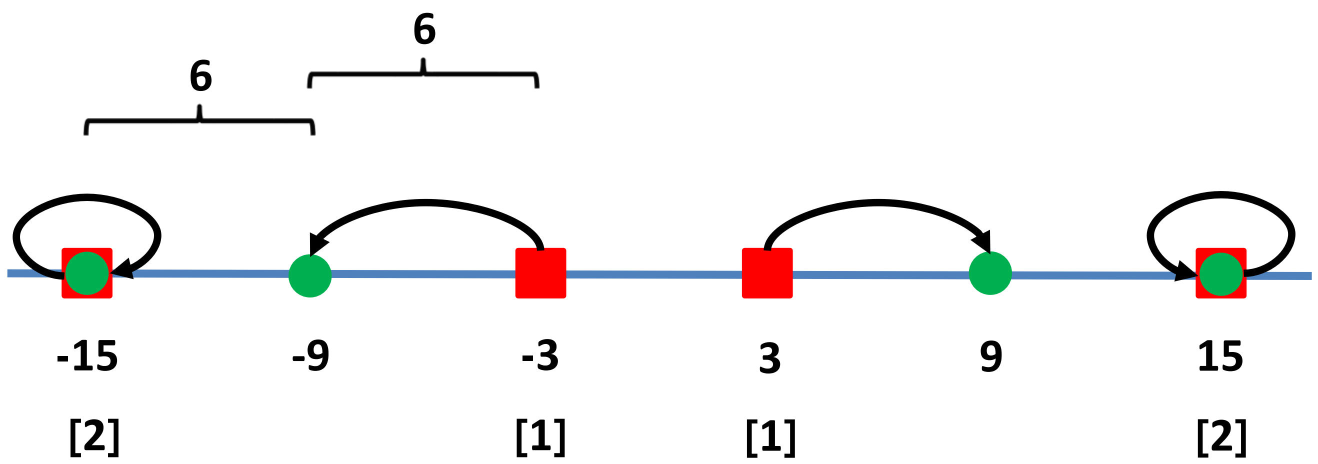
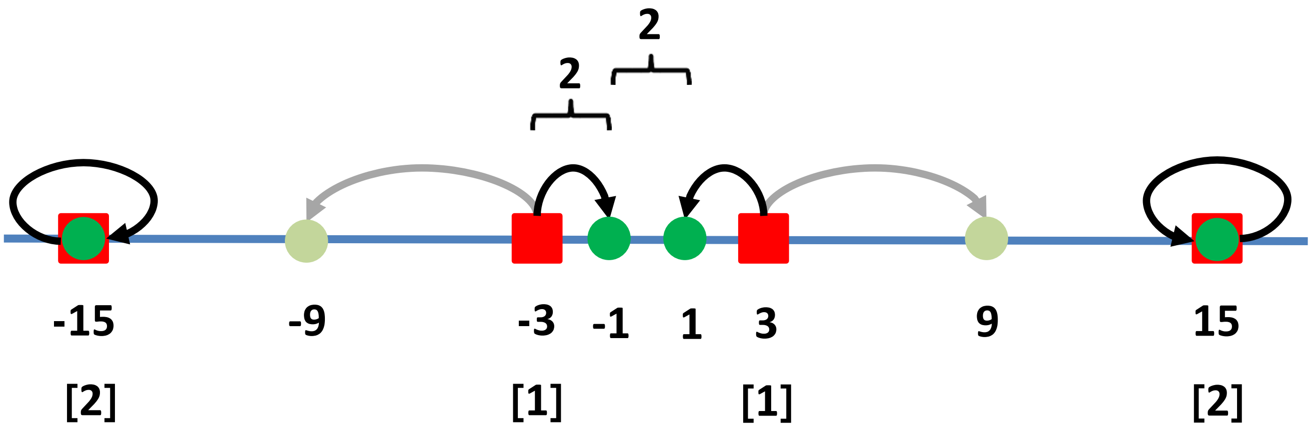


$\mathrm{SC}(\boldsymbol{s}, \boldsymbol{z}) = 12$

A lower bound on the price of anarchy

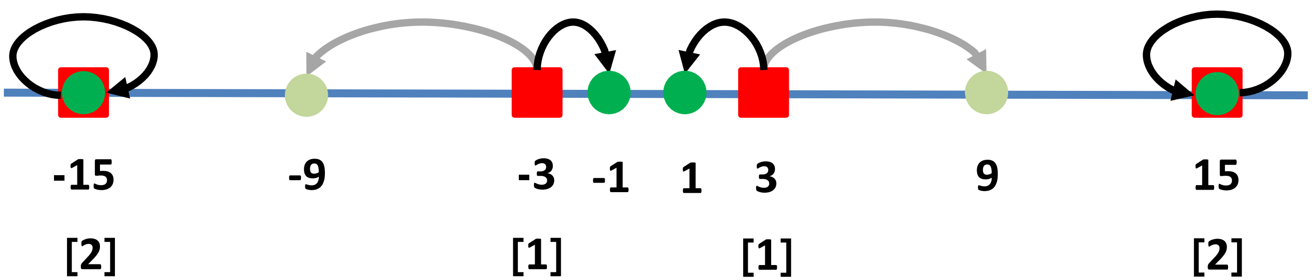
Theorem
For $k = 1$, the price of anarchy is at least 3



$\text{SC}(\boldsymbol{s}, \boldsymbol{z}') = 4$

A lower bound on the price of anarchy

<u>Theorem</u>
For $k = 1$, the price of anarchy is at least 3



$$\text{PoA} \geq \frac{\text{SC}(\boldsymbol{s}, \boldsymbol{z})}{\text{SC}(\boldsymbol{s}, \boldsymbol{z}')} = \frac{12}{4} = 3$$

□

Overview of results

- Pure equilibria may **not exist**, for any $k \geq 1$
- For $k = 1$, we can efficiently compute the best and the worst equilibrium
  - **Shortest and longest paths in DAGs**
- The price of anarchy and stability depend *linearly on* k
  - Proofs based on **LP duality** and **case analysis**
  - Tight bound of 3 on the price of anarchy for $k = 1$
  - Lower bounds on the mixed price of anarchy

Ownership transfer

Ownership transfer

- **Privatization of government assets**
  - Public electricity or water companies, airports, buildings, …
- **Sports tournaments organization**
  - World cup, Olympics, Formula 1, …

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  - Use of historical data related to the possible owners
  - Run an auction among the possible buyers

Ownership transfer

- **Privatization of government assets**
  - Public electricity or water companies, airports, buildings, …
- **Sports tournaments organization**
  - World cup, Olympics, Formula 1, …
- How should we decide who the new owner is going to be?
  - Use of historical data related to the possible owners
  - Run an auction among the possible buyers
- The new owner wants to maximize her own profit
  - Her decisions as the owner might critically affect the welfare of the society (company's employees and consumers, or the citizens)

Ownership transfer

- The goal is to make a decision that will sufficiently satisfy both the society and the new owner (if one exists)

Ownership transfer

- The goal is to make a decision that will sufficiently satisfy both the society and the new owner (if one exists)

- **Auction + expert advice**
  - The auction guarantees that the selling price is the best possible
  - The expert guarantees the well-being of the society

A simple model

- One item for sale
- **Two possible buyers A and B**
  - Each buyer i has a monetary valuation w_i for the item
- **One expert**
  - The expert has **von Neumann-Morgenstern** valuations $v(\cdot)$ for the three options:
    (1) sell the item to buyer A
    (2) sell the item to buyer B
    (3) Do not sell the item ($\emptyset$)
  - vNM valuations: [1, x, 0]

A simple model

- Design **mechanisms** that
  - incentivize the buyers and the expert to **truthfully report** their preferences, and
  - decide the option $i \in \{A, B, \emptyset\}$ that maximizes the **social welfare**

$$\mathrm{SW}(i) = \begin{cases} v(i) + \dfrac{w_i}{\max(w_A, w_B)}, & i \in \{A, B\} \\ v(\emptyset), & \text{otherwise} \end{cases}$$

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- **Combination of approximate mechanism design**
  - **with money** for the buyers (Nisan & Ronen, 2001)
  - **without money** for the expert (Procaccia & Tennenholtz, 2013)

Problem difficulty

- **Mechanism: given input by the buyers and the expert, choose the option that maximizes the social welfare**
  - Can this mechanism incentivize the participants to truthfully report their valuations?

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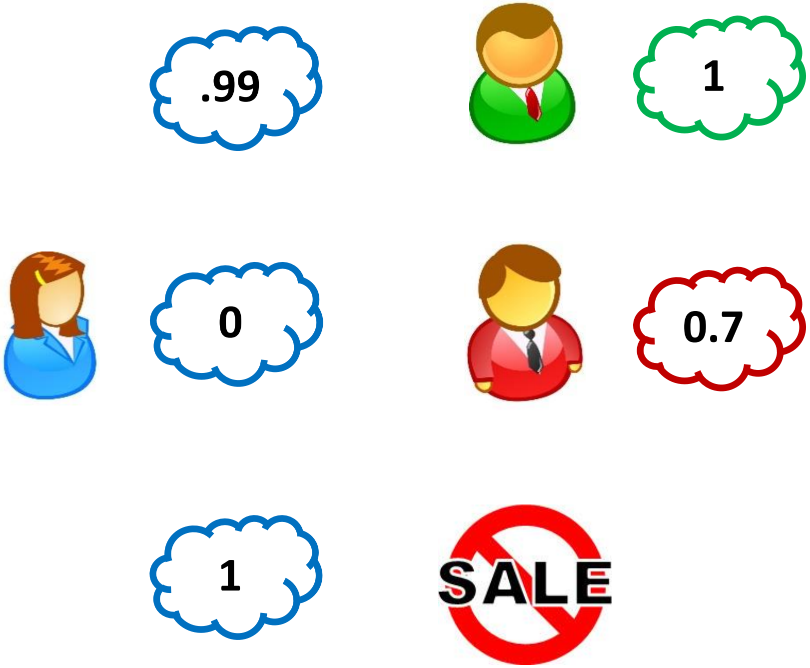
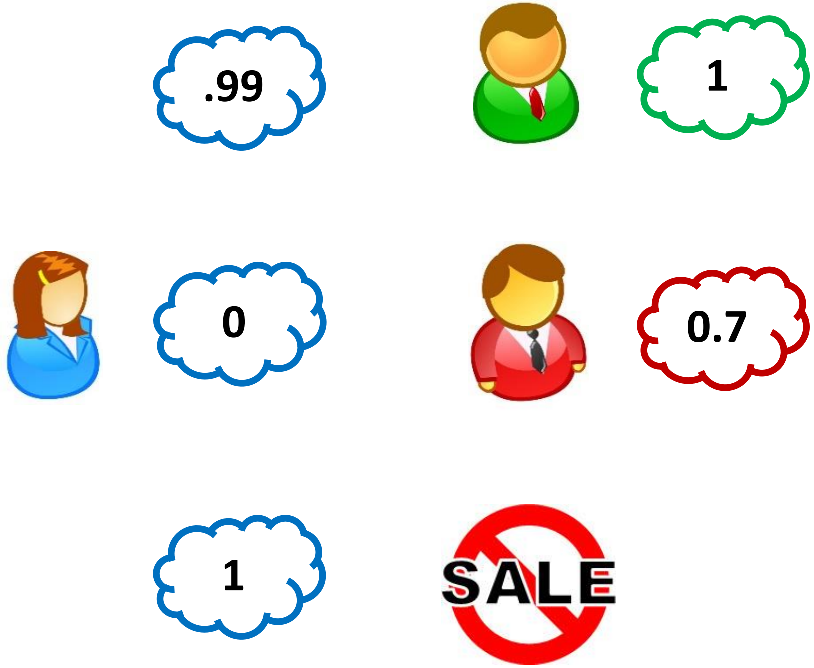
Examples of truthful mechanisms



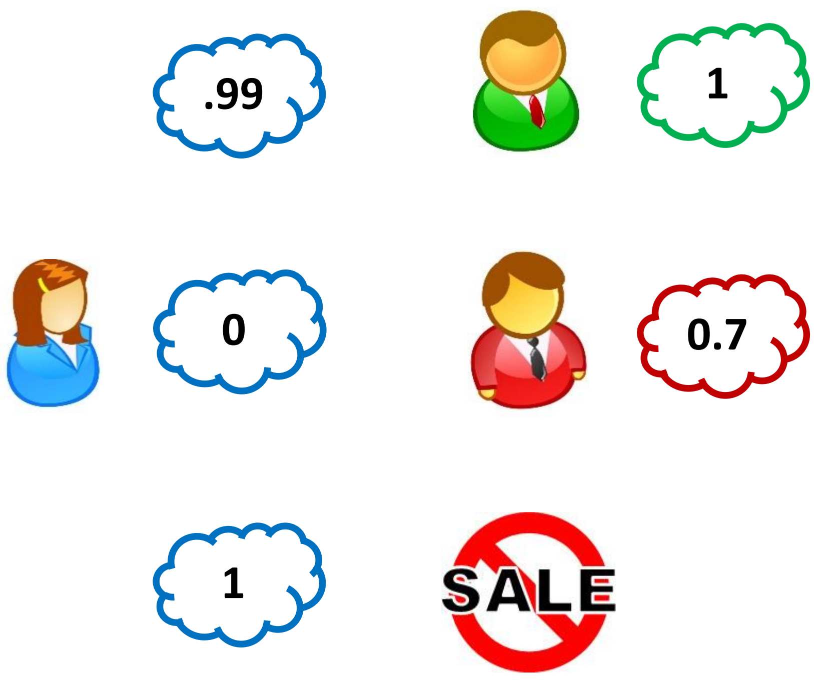
Examples of truthful mechanisms

- **Mechanism:** choose the favorite option of the expert



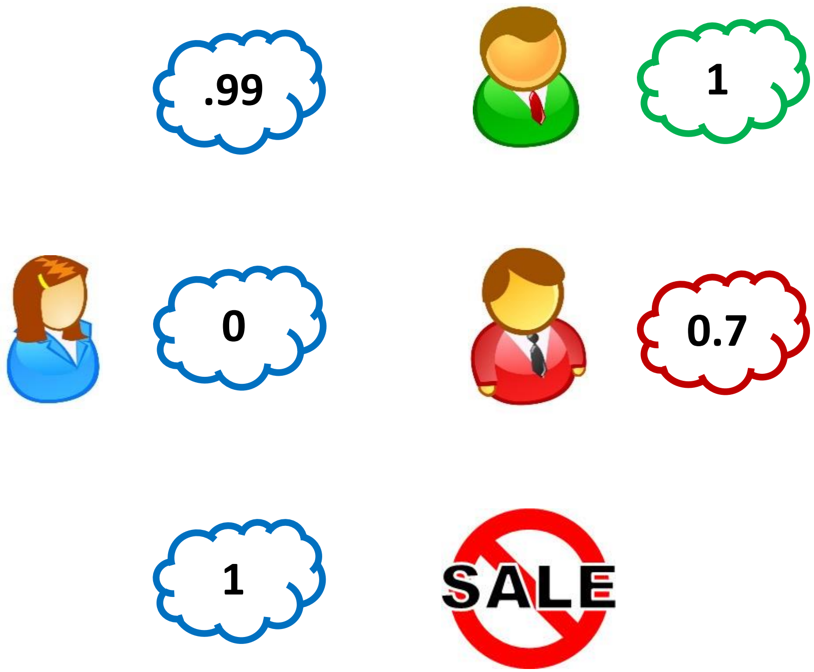
Examples of truthful mechanisms

- **Mechanism: choose the favorite option of the expert**
- SW(mechanism) = SW(no-sale) = 1 *vs.* SW(green) ≈ 2
  - approximation ratio = 2



Examples of truthful mechanisms

- **Mechanism: with probability 2/3 choose the expert's favorite option, and with probability 1/3 choose the expert's second favorite option**
- SW(mechanism) = SW(no-sale) · 2/3 + SW(green) · 1/3 ≈ 4/3
  - 3/2-approximate



Overview of results

class of mechanisms	approx
ordinal	1.5
bid-independent	1.377
expert-independent	1.343
randomized template	1.25
deterministic template	1.618
deterministic	$\geq$ 1.618
all	$\geq$ 1.14

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Mechanisms that base their decision only on the relative order of the values reported by the expert or the buyers

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Mechanisms that base their decision solely on the values reported by the expert

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Mechanisms that base their decision on the values reported by the expert *and* the buyers

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deterministic	≥ 1.618
all	≥ 1.14

Unconditional lower bounds for all mechanisms

Revenue maximization in combinatorial sales

The asymmetric binary matrix partition

- $\boldsymbol{A}$ = binary matrix with n rows and m columns

$\boldsymbol{A} =$

0	**0**	**0**	**1**
1	**0**	**0**	**0**
0	**0**	**1**	**0**
1	**0**	**1**	**1**

The asymmetric binary matrix partition

- $\boldsymbol{A}$ = binary matrix with n rows and m columns
- $\boldsymbol{p}$ = probability distribution over the columns of $\boldsymbol{A}$

$\boldsymbol{p} =$	10%	20%	25%	45%
	0	0	0	1
$\boldsymbol{A} =$	1	0	0	0
	0	0	1	0
	1	0	1	1

The asymmetric binary matrix partition

- $\boldsymbol{A}$ = binary matrix with n rows and m columns
- $\boldsymbol{p}$ = probability distribution over the columns of $\boldsymbol{A}$
- $\boldsymbol{B}$ = partition scheme
  - Consists of a partition B_i of the columns for every row i

$\boldsymbol{p} =$	**10%**	**20%**	**25%**	**45%**
	0	**0**	**0**	**1**
$\boldsymbol{A} =$	**1**	**0**	**0**	**0**
	0	**0**	**1**	**0**
	1	**0**	**1**	**1**

The asymmetric binary matrix partition

- $\boldsymbol{A^B}$ = smooth matrix that is the result of the application of the partition scheme $\boldsymbol{B}$ on matrix $\boldsymbol{A}$

$$j \in B_{ik} \implies A_{ij}^{B} = \frac{\sum_{\ell \in B_{ik}} p_\ell \cdot A_{i\ell}}{\sum_{\ell \in B_{ik}} p_\ell}$$

$\boldsymbol{p} =$

10%	20%	25%	45%

$\boldsymbol{A} =$

0	0	0	1
1	0	0	0
0	0	1	0
1	0	1	1

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- $\boldsymbol{A}^{\boldsymbol{B}}$ = smooth matrix that is the result of the application of the partition scheme $\boldsymbol{B}$ on matrix $\boldsymbol{A}$

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$\boldsymbol{p} =$	**10%**	**20%**	**25%**	**45%**
$\boldsymbol{A} =$	**0**	**0**	**0**	**1**
	1	**0**	**0**	**0**
	0	**0**	**1**	**0**
	1	**0**	**1**	**1**

$$A_{41}^{B} = \frac{10\% \cdot 1 + 20\% \cdot 0}{10\% + 20\%} = 0.33$$

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$\boldsymbol{A} =$	0	0	0	1
	1	0	0	0
	0	0	1	0
	1	0	1	1

$$A_{41}^{B} = \frac{10\% \cdot 1 + 20\% \cdot 0}{10\% + 20\%} = 0.33$$

$$A_{23}^{B} = \frac{10\% \cdot 1 + 20\% \cdot 0 + 25\% \cdot 0}{10\% + 20\% + 25\%} = 0.18$$

The asymmetric binary matrix partition

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$$j \in B_{ik} \implies A^B_{ij} = \frac{\sum_{\ell \in B_{ik}} p_\ell \cdot A_{i\ell}}{\sum_{\ell \in B_{ik}} p_\ell}$$

$\boldsymbol{p} =$ (header row), $\boldsymbol{A} =$ (body rows)

10%	20%	25%	45%
0	0	0	1
1	0	0	0
0	0	1	0
1	0	1	1

10%	20%	25%	45%
0	0.5	0.5	0.5
0.18	0.18	0.18	0
0.25	0.25	0.25	0.25
0.33	0.33	1	1

$= \boldsymbol{A^B}$

The asymmetric binary matrix partition

- Partition value of scheme $\boldsymbol{B}$:

$$v^{\boldsymbol{B}}(\boldsymbol{A},\boldsymbol{p}) = \sum_{j\in[m]} p_j \cdot \max_i A_{ij}^B$$

$\boldsymbol{p} =$

10%	20%	25%	45%
0	0	0	1
1	0	0	0
0	0	1	0
1	0	1	1

$\boldsymbol{A} =$

10%	20%	25%	45%
0	0.5	0.5	0.5
0.18	0.18	0.18	0
0.25	0.25	0.25	0.25
0.33	0.33	1	1

$= \boldsymbol{A}^{\boldsymbol{B}}$

The asymmetric binary matrix partition

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$$v^{\boldsymbol{B}}(\boldsymbol{A},\boldsymbol{p}) = \sum_{j\in[m]} p_j \cdot \max_i A_{ij}^{B}$$

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$\boldsymbol{A} =$

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0.25	0.25	0.25	0.25
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The asymmetric binary matrix partition

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$$v^{\boldsymbol{B}}(\boldsymbol{A},\boldsymbol{p}) = \sum_{j\in[m]} p_j \cdot \max_i A_{ij}^B$$

$\boldsymbol{p} =$

10%	20%	25%	45%
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0	0	1	0
1	0	1	1

$\boldsymbol{A} =$

10%	20%	25%	45%
0	0.5	0.5	0.5
0.18	0.18	0.18	0
0.25	0.25	0.25	0.25
0.33	0.33	1	1

$= \boldsymbol{A}^{\boldsymbol{B}}$

$$v^{\boldsymbol{B}}(\boldsymbol{A},\boldsymbol{p}) = (10\% \cdot 0.33) + (20\% \cdot 0.5) + (25\% \cdot 1) + (45\% \cdot 1) = 0.83$$

The asymmetric binary matrix partition

- **Objective:** Given $\boldsymbol{A}$ and $\boldsymbol{p}$, compute a partition scheme $\boldsymbol{B}$ with maximum value $v^{\boldsymbol{B}}(\boldsymbol{A}, \boldsymbol{p})$

The asymmetric binary matrix partition

- **Objective:** Given $\boldsymbol{A}$ and $\boldsymbol{p}$, compute a partition scheme $\boldsymbol{B}$ with maximum value $v^{\boldsymbol{B}}(\boldsymbol{A}, \boldsymbol{p})$

- **Application: Revenue maximization in take-it-or-leave-it sales**
  - There are m items and n possible buyers with valuations over the items
  - The seller has full information, while the buyers do not
  - How can the seller group the items and sell them to the buyers, in order to maximize her expected profit?

- **Asymmetric information** (Akerlof, 1970) (Crawford & Sobel, 1982) (Milgrom & Weber, 1982) (Ghosh et al., 2007) (Emek et al., 2012) (Miltersen & Sheffet, 2012)

Previous results

- Problem introduced by Alon, Feldman, Gamzu and Tennenholtz (2013)

- APX-hard
- 0.563-approximation algorithm for the case of uniform probability distributions
- 0.077-approximation algorithms for general distributions
- Other approximations for non-binary values

An improved approximation algorithm for uniform distributions

Greedy algorithm

- *Cover phase:* Compute a full cover of the one-columns (columns that contain at least one 1-value)
- *Greedy phase:* For each zero-column (containing only 0-values), add the column to the bundle that maximizes the column's marginal contribution to the partition value

25%	25%	25%	25%
1	1	0	0
1	1	0	0
0	1	0	0
1	0	0	0

An improved approximation algorithm for uniform distributions

Greedy algorithm

- *Cover phase:* Compute a full cover of the one-columns (columns that contain at least one 1-value)
- *Greedy phase:* For each zero-column (containing only 0-values), add the column to the bundle that maximizes the column's marginal contribution to the partition value

25%	25%	25%	25%
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0	1	0	0
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An improved approximation algorithm for uniform distributions

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- The marginal contribution of a zero-column when it is added to a bundle that already contains x zeros and y ones is:

$$\boldsymbol{\Delta(x, y) = (x+1)\frac{y}{x+y+1} - x\frac{y}{x+y}}$$

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GREEDY = 3/4

An improved approximation algorithm for uniform distributions

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25%	25%	25%	25%
1	1	0	0
1	1	0	0
0	1	0	0
1	0	0	0

GREEDY $= 3/4$

OPT $= 5/6$

$$\rho \geq \frac{\text{GREEDY}}{\text{OPT}} = \frac{9}{10}$$

Overview of results

- 0.9-approximation algorithm for uniform probability distributions
  - **Greedy** algorithm
  - Analysis using **linear programming** (factor-revealing LPs)
- 0.58-approximation algorithm for general probability distributions
  - Reduction to **submodular welfare maximization**

Papers in this thesis

- **The efficiency of resource allocation mechanisms for budget-constrained users**
  - I. Caragiannis and A. A. Voudouris
  - *Proceedings of the 19th ACM Conference on Economics and Computation (EC)*, pages 681-698, 2018

- **Bounding the inefficiency of compromise**
  - I. Caragiannis, P. Kanellopoulos, and A. A. Voudouris
  - *Proceedings of the 26th International Joint Conference on Artificial Intelligence (IJCAI)*, pages 142-148, 2017

Papers in this thesis

- **Truthful mechanisms for ownership transfer**
  - I. Caragiannis, A. Filos-Ratsikas, S. Nath, and A. A. Voudouris
  - Preliminary version to be presented at the *first Workshop on Opinion Aggregation, Dynamics, and Elicitation (WADE@EC18)*, 2018

- **Near-optimal asymmetric binary matrix partitions**
  - F. Abed, I. Caragiannis, and A. A. Voudouris
  - *Algorithmica*, vol. 80(1), pages 48-72, 2018
  - Extended abstract in *Proceedings of the 40th International Symposium on Mathematical Foundations of Computer Science (MFCS)*, pages 1-13, 2015

Other papers

- **Mobility-aware, adaptive algorithms for wireless power transfer in ad hoc networks**
  - A. Madhja, S. Nikoletseas, and A. A. Voudouris
  - *Proceedings of the 14th International Symposium on Algorithms and Experiments for Wireless Networks (ALGOSENSORS)*, 2018
- **Peer-to-peer energy-aware tree network formation**
  - A. Madhja, S. Nikoletseas, D. Tsolovos, and A. A. Voudouris
  - *Proceedings of the 16th ACM International Symposium on Mobility Managements and Wireless Access (MOBIWAC)*, 2018
- **Efficiency and complexity of price competition among single product vendors**
  - I. Caragiannis, X. Chatzigeorgiou, P. Kanellopoulos, G. A. Krimpas, N. Protopapas, and A. A. Voudouris
  - *Artificial Intelligence Journal*, vol. 248, pages 9-25, 2017
  - Extended abstract in *Proceedings of the 24th International Joint Conference on Artificial Intelligence (IJCAI)*, pages 25-31, 2015

Other papers

- **Optimizing positional scoring rules for rank aggregation**
  - I. Caragiannis, X. Chatzigeorgiou, G. A. Krimpas, and A. A. Voudouris
  - *Proceedings of the 31st AAAI Conference on Artificial Intelligence (AAAI)*, pages 430-436, 2017
- **How effective can simple ordinal peer grading be?**
  - I. Caragiannis, G. A. Krimpas, and A. A. Voudouris
  - *Proceedings of the 17th ACM Conference on Economics and Computation (EC)*, pages 323-340, 2016
- **co-rank: an online tool for collectively deciding efficient rankings among peers**
  - I. Caragiannis, G. A. Krimpas, M. Panteli, and A. A. Voudouris
  - *Proceedings of the 30th AAAI Conference on Artificial Intelligence (AAAI)*, pages 4351-4352, 2016

Other papers

- **Welfare guarantees for proportional allocations**
  - I. Caragiannis and A. A. Voudouris
  - Theory of Computing Systems, vol. 59(4), pages 581-599, 2016
  - Extended abstract in *Proceedings of the 7th International Symposium on Algorithmic Game Theory (SAGT)*, pages 206-217, 2014
- **Aggregating partial rankings with applications to peer grading in massive online open courses**
  - I. Caragiannis, G. A. Krimpas, and A. A. Voudouris
  - *Proceedings of the 14th International Conference on Autonomous Agents and Multi-Agent Systems (AAMAS)*, pages 675-683, 2015

Thank you!